On a non-linear dynamic solvency control model

Malinovskii Vsevolod

Steklov Mathematical Institute

119991, Gubkina Str., 8, Moscow, Russia

Tel.: (095) 135 22 91

E-mail: malinov@orc.ru, malinov@mi.ras.ru

Abstract

A dynamic control model of the insurance process over n successive accounting years is considered. The analytical inference about the model requires investigations of a class of kernels describing yearly insurance mechanism.

Aiming the kernels, the approximations for the distribution of the risk reserve at time t conditional on ruin within time t in the Andersen's collective risk model are obtained. Corrected approximations for the mean and certain numerical results are also presented.

Keywords: dynamic solvency, control, finite time ruin, approximations

1. A dynamic control model

In the paper [**3**] Harald Cramér responded to certain criticism on the Lundberg theory. He wrote: "In view of certain misconceptions that have appeared it is, however, necessary to point out that Lundberg repeatedly emphasizes the practical importance of some arrangement which automatically prevents the risk reserve from growing unduly. This point is, in fact, extensively discussed in the papers of 1909, 1919 and 1926–1928. One possible arrangement proposed to this end is to work with a security factor $\tau = \tau(x)$ which is a decreasing function of the risk reserve $R(t) = x$. Another possibility is to dispose, at predetermined epochs, of part of the risk reserve for bonus distribution. By either method, the growth of the risk reserve may be efficiently controlled."

The subject of this paper is the latter arrangement, for which we introduce a class of models of the insurance process over n successive accounting years ("predetermined epochs" as in the citation above) which diagram is

$$w_0 \underbrace{\xrightarrow{q_0} u_0 \xrightarrow{g_1} w_1}_{\text{1-st year}} \cdots \xrightarrow{g_{k-1}} w_{k-1} \underbrace{\xrightarrow{q_{k-1}} u_{k-1} \xrightarrow{g_k} w_k}_{k\text{-th year}} \cdots .$$

In words, at the end of each $(k-1)$-th year, $k = 1, 2, \ldots$, at time $t^{\dagger}_{k-1}$, the aggregate state of the insurer w_{k-1} is observed; at the beginning of the k-th year, at time t^{*}_{k}, the control variable u_{k-1} is picked up according to the rule q_{k-1}. It actuates the probabilistic mechanism of insurance denoted by g_k and produces the aggregate state variable w_k describing the insurer position at the end of the k-th insurance year, at time $t^{\dagger}_{k}$, and so on. The models of this type, mainly linear, were considered by a range of authors (see e.g., [**5**], [**12**] and references therein).

In our particular model the yearly account of the company is supposed to consist of two items: (i) the size of the risk reserve at the end of the insurance year, and (ii) the value indicating a fall of the risk reserve below a level $z \geqslant 0$. Thus, the aggregate state variable is a two-dimensional random vector. The risk reserve at the end of the insurance year is its first component, the indicator of the event above is the second one. Let the control q_{k-1} consists in adjustment of the capital at the beginning of each insurance year t^{*}_{k}, $k = 1, 2, \ldots, n$. Increase may be interpreted e.g., as dividend payment (bonus as in the citation above), decrease — as external capital borrowing.

The aggregate variables W_k range over the state space $(\mathrm{W}, \mathcal{W})$ which is Cartesian product of the real line R and of the two-point space $\{0,1\}$, endowed with a σ-field, and the control space $(\mathrm{U}, \mathcal{U})$ is the positive half-line R^+, endowed with a σ-field.

It comes natural to assume that the first component of $W_k = (W_{1,k}, W_{2,k})$ is governed by the linear equation

$$W_{1,k} = U_{k-1} + \xi_k, \tag{1}$$

where $U_{k-1} = q_{k-1}(W_{k-1}) \in \mathrm{U}$. For simplicity we bound ourselves by nonrandomized and Markovian controls $q_{k-1} : \mathrm{R} \times \{0,1\} = \mathrm{W} \to \mathrm{U} = \mathrm{R}^+$ and take

$$\xi_k = I(t_k) - S(t_k), \tag{2}$$

where $t_k = t_k^\dagger - t_k^*$. Adopting the Lundberg collective approach and switching to the operational time, the k-th year total premium income is $I(t_k) = c_k t_k$, with the premium rate c_k constant over this year, and the k-th year total claims outcome is $S(t_k) = \sum_{i=1}^{N(t_k)} Y_i$ (see definition of $N(t)$ and Y_i in the next section), so that $W_{1,k} = R(t_k)$, where

$$R(t) = U_{k-1} + c_k t - S(t), \quad 0 < t \leqslant t_k. \tag{3}$$

For the second component of $W_k = (W_{1,k}, W_{2,k})$, introduce $M(t) = \inf_{0<s\leqslant t} R(s)$, where $0 < t \leqslant t_k$, and $W_{2,k} = \mathbf{1}_{\{M(t_k)<z\}}$.

The probabilistic mechanism of insurance on the k-th successive year is defined by

$$\begin{aligned} g_k(dw_{1,k} \times 0 \mid u_{k-1}) &= \mathbf{P}\big\{R(t_k) \in dw_{1,k}, M(t_k) \geqslant z \mid U_{k-1} = u_{k-1}\big\}, \\ g_k(dw_{1,k} \times 1 \mid u_{k-1}) &= \mathbf{P}\big\{R(t_k) \in dw_{1,k}, M(t_k) < z \mid U_{k-1} = u_{k-1}\big\}. \end{aligned} \tag{4}$$

Addressing to the control q_{k-1}, any fall of the insurer's capital below a certain level z might be declared a worrying event indicating aggravation of the financial solvency situation. An oversimplified, but sensible example expressing this worry is the control

$$\begin{aligned} q_{k-1}(w_{1,k-1} \times 1) &= (1-\alpha) w_{1,k-1} \mathbf{1}_{\{w_{1,k-1} \geqslant y\}} \\ &\qquad + w_{1,k-1} \mathbf{1}_{\{z < w_{1,k-1} < y\}} + z \mathbf{1}_{\{w_{1,k-1} \leqslant z\}}, \\ q_{k-1}(w_{1,k-1} \times 0) &= (1-\alpha) w_{1,k-1}, \end{aligned} \tag{5}$$

where $0 < z < y$ and $0 \leqslant \alpha \leqslant 1$. According to (5), in the "worrying" situation (i.e., $W_{2,k-1} = 1$) the $(k-1)$-th year-end dividends equal to α-th fraction of the $(k-1)$-th year-end capital $W_{1,k-1}$ are paid out if and only if this capital exceeds the level y; if the year-end capital is less than z, borrowing is applied to match this level. In the "regular" case (i.e., $W_{2,k-1} = 0$, which entails $W_{1,k-1} \geqslant z$) the dividends are payed out regardless the capital size.

For a fixed initial distribution $g_0(w_0) = \mathbf{P}\{W_0 \in dw_0\}$ the functions (4) and the control (5) specify on the probability space $(\Omega, \mathcal{F}, \mathbf{P})$ a random sequence (W_k, U_k), $k = 0, 1, \dots,$ which determines the dynamic control insurance model. Combine (5) and (4): for $w_{k-1}, w_k \in \mathrm{W} = \mathrm{R} \times \{0, 1\}$ we obtain

$$p_k(dw_k \mid w_{k-1}) = g_k(dw_k \mid q_{k-1}(w_{k-1})), \tag{6}$$

and for the strategy $\gamma_n = \{q_i(\cdot), i = 0, 1, \dots, n-1\}$

$$\mathbf{P}^\gamma\{W_0 \in A_0, \dots, W_n \in A_n\} = \int_{A_0} p_0(dw_0) \int_{A_1} p_1(dw_1 \mid w_0) \dots \int_{A_n} p_n(dw_n \mid w_{n-1}), \tag{7}$$

where $A_k \in \mathcal{W}$, $k = 0, 1, \dots, n$. Furthermore, for an integrable cost functional F

$$S(F; \gamma_n) = \mathbf{E}^\gamma F(W_0, \dots, W_n) = \int_{\mathrm{W}} p_0(dw_0) \dots \int_{\mathrm{W}} p_n(dw_n \mid w_{n-1}) F(w_0, \dots, w_n). \tag{8}$$

In particular, the choice $F(w_0, \dots, w_n) = (1-\alpha) \sum_{i=0}^n (\mathbf{1}_{\{w_{2,i}=1, w_{1,i} \geqslant y\}} + \mathbf{1}_{\{w_{2,i}=0\}}) w_{1,i}$ yields the total dividends, and the choice $F(w_0, \dots, w_n) = \sum_{i=0}^n (z - w_{1,i}) \mathbf{1}_{\{w_{1,i} \leqslant z\}}$ yields the total borrowing up to the n-th year under the control (5). Note that inflation and interests are easy to incorporate into these year-by-year sums.

It is a customary practice to approach the problem of optimization of (8) over a class of admissible strategies $\mathfrak{F}$,

$$S(F) = \min_{\gamma_n \in \mathfrak{F}} S(F; \gamma_n), \tag{9}$$

following the lines e.g., of [**9**], [**10**] though according to comment of S. Benjamin on the paper by K. Borch [**2**], there is a danger of not running anything well aiming at "the best", with too much "optimizing".

We are not interested here in asymptotics, as n increases, rather in analytical calculation of (7) and (8) given a strategy γ_n, as n is fixed. Considerable technical difficulty of such problems together with their practical importance called forth a wide application of simulation analysis (see [**4**]). However, according to [**11**], "as a compromise between simulation and analytical methods it may be advisable to perform some simplified calculations analytically first". Bearing in mind this interest, we consider further on some analytical results for the kernels (4), where $u_{k-1} \geqslant (1-\alpha)z$, as z is sufficiently large.

We formulate these results in the standard framework of the ruin theory, investigating correlation between ruin within time t and the insurer's surplus at time t in Andersen's model with light tailed claims and positive safety loading. Incidentally, we generalize the classical Cramér–Lundberg approximation (see e.g., [**1**]) for the finite time ruin probability $\psi(t,u)$,

$$\lim_{u\to\infty}\sup_{t\geqslant 0}\left|e^{\varkappa u}\psi(t,u)-C\Phi_{(m_1u,D_1^2u)}(t)\right|=0,\quad\text{as}\quad u\to\infty. \tag{10}$$

2. Further notation and assumptions

Being in the framework of Andersen's risk model, recall that it comes from the i.i.d. random vectors $\{(Y_i,T_i)\}_{i\geqslant 1}$, where T_i are the interclaim times and Y_i are the amounts of claims, with the probability distribution function (p.d.f.) $B_{Y,T}(y,t)=\mathbf{P}\{Y_1\leqslant y,T_1\leqslant t\}$ and the characteristic function (ch.f.) $\beta_{Y,T}(t_1,t_2)=\mathbf{E}\exp(it_1Y_1+it_2T_1)$. These random vectors generate the risk reserve process

$$R_u(t)=u+ct-\sum_{i=1}^{N(t)}Y_i,\quad t\geqslant 0, \tag{11}$$

where $u>0$ is the initial risk reserve, $c>0$ is the risk premium rate, and $N(t)$ is the number of claims up to time t, i.e. the largest n for which $\sum_{i=1}^n T_i\leqslant t$ (we put $N(t)=0$ if $T_1>t$). Consider

$$c=(1+\tau)\mathbf{E}Y_1/\mathbf{E}T_1, \tag{12}$$

with τ called relative safety loading and assume $\tau>0$. Evidently, the equality (12) is equivalent to $\tau=c\mathbf{E}T_1/\mathbf{E}Y_1-1$.

Ruin occurs at time s as $R_u(s)<0$ and the probability that ruin occurs within the time interval $(0,t]$ is $\psi(t,u)=\mathbf{P}[\inf_{0<s\leqslant t}R_u(s)<0]$.

For $i=1,2,\ldots$ introduce i.i.d. random variables $X_i=Y_i-cT_i$ and put $S_n=\sum_{i=1}^n X_i$, $V_n=\sum_{i=1}^n Y_i$. For the p.d.f. $B(x,y)=\mathbf{P}\{X_1\leqslant x,T_1\leqslant y\}$ and for a positive solution $\varkappa$ of the Lundberg equation,

$$\mathbf{E}\exp(\varkappa X_1)=1, \tag{13}$$

introduce an associate p.d.f. by $\overline{B}(dx,dy)=e^{\varkappa x}B(dx,dy)$. For notational convenience, introduce the associated sequence $\{(\overline{X}_i,\overline{T}_i)\}_{i\geqslant 1}$ of i.i.d. random vectors having the p.d.f. $\overline{B}(x,y)$, and $\overline{S}_n=\sum_{i=1}^n\overline{X}_i$, $\overline{U}_n=\sum_{i=1}^n\overline{T}_i$. Put

$$\nu^{i,j}=\mathbf{E}Y_1^iT_1^j,\quad\overline{\nu}^{i,j}=\mathbf{E}\overline{X}_1^i\overline{T}_1^j,\quad i,j=0,1\ldots\ . \tag{14}$$

Introduce

$$
\begin{aligned}
m_1 = \overline{\nu}^{0,1}/\overline{\nu}^{1,0}, &\quad m_2 = \tau\nu^{1,0}/\nu^{0,1},\\
D_1^2 &= ((\overline{\nu}^{0,1})^2\overline{\nu}^{2,0} - 2\overline{\nu}^{1,0}\overline{\nu}^{0,1}\overline{\nu}^{1,1} + (\overline{\nu}^{1,0})^2\overline{\nu}^{0,2})/(\overline{\nu}^{1,0})^3,\\
D_2^2 &= ((\nu^{0,1})^2\nu^{2,0} - 2\nu^{1,0}\nu^{0,1}\nu^{1,1} + (\nu^{1,0})^2\nu^{0,2})/(\nu^{0,1})^3,\\
C &= \frac{1}{\varkappa\overline{\nu}^{1,0}}\exp\left(-\sum_{n=1}^{\infty}\frac{1}{n}\big[\mathbf{P}(S_n > 0) + \mathbf{P}(\overline{S}_n \leqslant 0)\big]\right),
\end{aligned}
\tag{15}
$$

and for the Normal distribution and density functions $\Phi_{(\mu,\sigma^2)}(z)$ and $\varphi_{(\mu,\sigma^2)}(z)$ introduce

$$
g(z) = z + \varphi_{(0,1)}(z)\Phi^{-1}_{(0,1)}(z). \tag{16}
$$

Using the Mill's relation, we have $g(z) = z(1 + \overline{o}(1))$, as $z \to \infty$, and using the approximation $\Phi_{(0,1)}(z) = \frac{1}{2} + \varphi_{(0,1)}(z)(z + \frac{1}{3}z^3 + \dots)$, we have $g(z)$ approximated by $(1 - 4\varphi^2_{(0,1)}(z))z + 2\varphi_{(0,1)}(z)$ for z in a neighborhood of zero.

3. Approximations

Introduce

$$
\psi(w, z; t, u) = \mathbf{P}\big[R_u(t) \leqslant w, \inf_{0<s\leqslant t} R_u(s) < z\big], \quad \psi(w; t, u) = \psi(w, 0; t, u). \tag{17}
$$

Evidently, $\psi(+\infty; t, u) = \psi(t, u)$, for c constant $\psi(w, z; t, u) = \psi(w - z; t, u - z)$, and the kernel $\psi(w, z; t, u)$ is closely related with (4).

THEOREM 1. *Suppose that in the collective risk model with $\tau > 0$ the characteristic function $\beta_{Y,T}(t_1, t_2)$ is absolutely integrable and $0 < D_1, D_2 < \infty$. Then, as $u \to \infty$,*

$$
\lim_{u\to\infty}\sup_{t\geqslant 0, w\in\mathbf{R}}\left|e^{\varkappa u}\psi(w; t, u) - C\int_0^t \varphi_{(m_1 u, D_1^2 u)}(z)\Phi_{(m_2[t-z], D_2^2[t-z])}(w)\,dz\right| = 0. \tag{18}
$$

The approximation (10) is a particular case of (18).

THEOREM 2. *Under the conditions of Theorem 1*

$$
\begin{aligned}
\mathbf{E}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big] &= m_2 D_1\sqrt{u}\, g\left(\frac{t - m_1 u}{D_1\sqrt{u}}\right)(1 + \overline{o}(1)),\\
\mathbf{D}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big] &= D_2^2 D_1\sqrt{u}\, g\left(\frac{t - m_1 u}{D_1\sqrt{u}}\right)(1 + \overline{o}(1)),
\end{aligned}
$$

as $u \to \infty$.

REMARK 1. In the particular case of the Poisson/Exponential model the approximations are easy to express in terms of intensities. Assume that the (i.i.d.) amounts of claims $\{Y_i\}_{i\geqslant 1}$ and the (i.i.d.) inter-occurrence times $\{T_i\}_{i\geqslant 1}$ are mutually independent and exponential with parameters $\mu > 0$ and $\lambda > 0$ respectively. Then

$$c = \lambda(1+\tau)/\mu \tag{19}$$

and (compare to e.g., (2.5)–(2.7) in [**8**])

$$\begin{aligned} &\varkappa = \mu\tau/(1+\tau), \quad m_1 = \mu/(\lambda\tau(1+\tau)), \quad m_2 = \tau\lambda/\mu, \\ &D_1^2 = 2\mu/(\lambda^2\tau^3), \quad D_2^2 = 2\lambda/\mu^2, \quad C = 1/(1+\tau). \end{aligned} \tag{20}$$

In particular, the approximation for the expectation $\mathbf{E}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big]$ at the time point $t = m_1 u$ is

$$\sqrt{2u}g(0)\,/\sqrt{\mu\tau}. \tag{21}$$

4. Corrected approximation and a numerical example for the mean

Denote the ladder index $\mathcal{N} = \inf\{n : \overline{S}_n > 0\}$, the ladder height $\mathcal{H} = \overline{S}_{\mathcal{N}}$ and the ladder time point $\mathcal{T} = \overline{U}_{\mathcal{N}}$ and put $\mathcal{W} = \mathbf{E}(\mathcal{T}\mathbf{E}\mathcal{H} - \mathcal{H}\mathbf{E}\mathcal{T})$. Introduce

$$\begin{aligned} &\theta_1 = \frac{\mathbf{E}\mathcal{H}}{1-\mathbf{E}e^{-\varkappa\mathcal{H}}} - \frac{1}{\varkappa}, \quad \theta_2 = \frac{\mathbf{E}\mathcal{T}}{1-\mathbf{E}e^{-\varkappa\mathcal{H}}} - \frac{\mathbf{E}\mathcal{T}e^{-\varkappa\mathcal{H}}}{1-\mathbf{E}e^{-\varkappa\mathcal{H}}}, \\ &\theta_3 = \frac{1}{\varkappa} - \frac{\mathbf{E}\mathcal{H}e^{-\varkappa\mathcal{H}}}{1-\mathbf{E}e^{-\varkappa\mathcal{H}}}, \quad k_1 = \mathbf{E}\mathcal{W}^2, \quad k_2 = \frac{\mathbf{E}\mathcal{W}^3}{6k_1}, \\ &k_3 = \mathbf{E}\mathcal{T}\mathbf{D}\mathcal{H} - \mathbf{E}\mathcal{H}\mathbf{Cov}(\mathcal{H},\mathcal{T}). \end{aligned} \tag{22}$$

The following approximation elaborates the first relation of Theorem 2.

THEOREM 3. *Suppose that in the collective risk model with $\tau > 0$ the characteristic function $\beta_{Y,T}(t_1,t_2)$ is absolutely integrable, $0 < D_1, D_2 < \infty$ and $\mathbf{E}T_1^3 < \infty$. Suppose that the premium rate is c as in (12). Then, as $u \to \infty$,*

$$\begin{aligned} &\sup_{t\geqslant 0}\Big|\mathbf{E}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big]\psi(t,u) \\ &- \nu^{1,0}\left(1 - \frac{\nu^{0,2}}{2(\nu^{0,1})^2}\right)\psi(t,u) \\ &- Ce^{-\varkappa u}\tau\frac{\nu^{1,0}}{\nu^{0,1}}D_1\sqrt{u}\left[\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\Phi_{(0,1)}\left(\frac{t-m_1u}{D_1\sqrt{u}}\right) + \varphi_{(0,1)}\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\right] \\ &- Ce^{-\varkappa u}\tau\frac{\nu^{1,0}}{\nu^{0,1}}\Phi_{(0,1)}\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\left(\frac{k_3}{2(\mathbf{E}\mathcal{H})^2} + 3\frac{k_2}{\mathbf{E}\mathcal{H}} - \theta_1\frac{\mathbf{E}\mathcal{T}}{\mathbf{E}\mathcal{H}} + \theta_2\right) \end{aligned}$$

$$- Ce^{-\varkappa u}\tau\frac{\nu^{1,0}}{\nu^{0,1}}\Phi_{(0,1)}\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\left(\theta_2-\theta_1\frac{\mathbf{E}\mathcal{T}}{\mathbf{E}\mathcal{H}}-\frac{k_3}{(\mathbf{E}\mathcal{H})^2}-\theta_3\frac{\mathbf{E}T_1}{\tau\mathbf{E}Y_1}\right)$$
$$- Ce^{-\varkappa u}\tau\frac{\nu^{1,0}}{\nu^{0,1}}\varphi_{(0,1)}\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\left(\frac{t-m_1u}{D_1\sqrt{u}}\right)\left(\theta_1\frac{\mathbf{E}\mathcal{T}}{\mathbf{E}\mathcal{H}}-\theta_2-2\frac{k_2}{\mathbf{E}\mathcal{H}}\right)\Bigg| = \overline{o}(e^{-\varkappa u}).$$

Numerical example. Assume that the (i.i.d.) amounts of claims $\{Y_i\}_{i\geqslant 1}$ and the (i.i.d.) inter-occurrence times $\{T_i\}_{i\geqslant 1}$ are mutually independent and exponential with parameters $\mu > 0$ and $\lambda > 0$ respectively.

Lengthy but straightforward calculations similar to those described in Theorem 2 and Lemma 1 of [**6**] (see also pp. 890–891 and p. 907 of [**8**]) applied to Theorem 3 yield the following approximation for $\mathbf{E}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big]\psi(t,u)$ at the time point $t = m_1u$ with m_1 from (20):

$$Ce^{-\varkappa u}\Phi_{(0,1)}(0)\left(\frac{\sqrt{2u}}{\sqrt{\mu\tau}}g(0)-\frac{3+3\tau+\tau^2}{\mu(1+\tau)}\right). \tag{23}$$

The approximation at the time point $t = m_1u$ for $\psi(t,u)$,

$$Ce^{-\varkappa u}\Phi_{(0,1)}(0)\left(1-Q_1(0)\frac{\lambda\tau^{3/2}}{\sqrt{2\mu u}}g(0)\right), \tag{24}$$

where

$$Q_1(0) = \frac{2+\tau^2}{\lambda\tau(1+\tau)} - \frac{\tau+2}{2\lambda^2\tau^2},$$

is a corollary of Theorem 1 of [**6**]. For the expectation $\mathbf{E}\big[R_u(t) \mid \inf_{0<s\leqslant t} R_u(s) < 0\big]$ at the time point $t = m_1u$ these approximations yield

$$\left(\frac{\sqrt{2u}}{\sqrt{\mu\tau}}g(0)-\frac{3+3\tau+\tau^2}{\mu(1+\tau)}\right)\Bigg/\left(1-Q_1(0)\frac{\lambda\tau^{3/2}}{\sqrt{2\mu u}}g(0)\right). \tag{25}$$

Compare the approximation (21) and the corrected approximation (25) to the simulation results. For this end, simulate N risk reserve trajectories and calculate the mean value of the risk reserve at the time point $t = m_1u$ over those among them which fall below zero at least once within time $t = m_1u$. We report the results in the tables below omitting fractional parts. It is seen that the accuracy of the approximation (25) appears better than of (21). For further improvements one has to calculate more terms in the expansions (23) and (24).

The data in this table demonstrates a reasonably good accuracy. The poorer accuracy in the following table is due to a smaller τ. Calculation of more correction terms which

Table 1. $\lambda = \mu = 1,\ t = 99502,\ u = 500,\ \tau = 0.005,\ N = 10000.$

	Simulation runs							
	1	2	3	4	5	6	7	8
Number of trajectories which fall below zero	287	327	325	315	296	278	286	311
Empirical mean conditioned by zero	209	224	242	220	214	207	195	222
Approximation (21) for the mean	357							
Corrected approximation (25) for the mean	261							

are of a smaller order as u grows, but are increasing as τ decreases, becomes here more important.

Table 2. $\lambda = \mu = 1,\ t = 499500,\ u = 500,\ \tau = 0.001,\ N = 1000.$

	Simulation runs							
	1	2	3	4	5	6	7	8
Number of trajectories which fall below zero	189	213	190	222	184	227	396	397
Empirical mean conditioned by zero	326	369	346	339	368	358	310	335
Approximation (21) for the mean	798							
Corrected approximation (25) for the mean	442							

REMARK 2. To make the results of Theorem 1 more suitable for calculation of (7) and (8) the non-uniform bounds instead of merely uniform ones should be obtained. Otherwise, bounds on large deviations as in [**7**] are advisable. These results require the similar technique and will be published elsewhere.

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