Dr. V. K. Malinovskii

STEKLOV MATHEMATICAL INSTITUTE

E-mail: malinov@vsevo.mian.su
malinov@mi.ras.ru

Gubkina str., 8, 117966, Moscow
GSP-1, Russia

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Calculation of the ruin probabilities

Course description

After a review of the Andersen's collective risk model with a special emphasis on the Poisson/Exponential, or a "classical", case a technique to derive exact formulae for the probabilities of ruin within finite and infinite time will be introduced. Exact formulae for the probabilities of ruin will be derived in the "classical" case as a particular case of the general technique. The general case will require numerical inversions of the Laplace transformations. As a first alternative to the exact formulae, simulation technique will be discussed. As a second alternative, approximate formulae (as the initial capital grows) for the probabilities of ruin will be discussed. Emphasis will be put, on one hand, on the applicability of the formulae to problems occurring in the practical work of an actuary (with special emphasis on non-Poissonian claims arrivals) and, on the other hand, on the mathematical foundations of the formulae. Typically, problems of the following type will be considered: given certain data, what kind of numerical approach could be applied to derive the probability of ruin? Is it reasonable to apply an approximation? If yes, what will be accuracy of this approximation? Could an exact numerical technique, or a simulation technique, be applied and what could be the advantages or disadvantages compared to the approximations?

Program of the course

1. Collective risk models: Andersen's and Cramér risk model, "classical" P/E risk model. Probabilities of ruin. Poisson and renewal processes as claim arrival processes. Ref: 1, 2, 4, 7, 11, 12.
2. An overview of different approaches to calculation of the ruin probability within finite time: exact formulas, numerical procedures, approximations, simulation. Ref: 1, 2, 3, 4, 5, 8, 13, 14, 19, 20.
3. Several examples of claim arrival processes with heavy tailed interclaim times: Pareto, Kummer. The impact on the probabilities of ruin within finite time. Ref: 1, 2, 3, 4, 5, 8, 13, 14, 19, 20.
4. Exact formulae for the probabilities of ruin within finite time.
   (2a) "Classical" risk model: an exact formula.

(2b) Poisson claims arrivals and non-exponential claim severities.
(2c) Renewal claims arrivals and exponential claim amounts.
Ref: 7, 15, 16, 19.

5. A numerical technique: simulation (Siegmund's importance sampling, Pentikäinen's bunches of trajectories).
Ref: 3, 8, 14, 20.
6. A random walk approach to the problem of calculation of the probability of ruin within finite time. Associate random walks, ladder random variables, von Bahr's representation.
Ref: 1, 2, 6, 9.
7. Approximations (as the initial capital grows) for the probabilities of ruin (Cramér-Lundberg's approximation, von Bahr's formulae, asymptotic expansions). The sketch of the proof using renewal approach and CLT in high dimensions.
Ref: 2, 5, 6, 9, 13.
8. More details on alternative representations of asymptotic results (Normal-type approximations and expansions) through the cumulants of the ladder r.v. and through the Spitzer's sums. Representation of the approximations through the initial intensities in case of P/E model. Some technical aspects of calculations involving e.g., incomplete Beta functions.
Ref: 5, 13, 16.
9. Approximations and non-Poissonian claims arrivals: some traps for statistical implementation.
Ref: 3, 14.
10. Dynamic insurance model. The importance of the probability of ruin within finite time for calculation of reserves.
Ref: 17.
11. Some aspects of the risk model when the relative safety loading tends to zero as function of the growing initial capital. Approximations and sketch of the proof of the approximation. Expansions of the basic constants involving Bürmann–Lagrange theorem.
Ref: 15, 16.

References

The following abbreviations are used: *AS = Annales of Statistics*; *IME = Insurance: Mathematics and Economics*; *SAJ = Scandinavian Actuarial Journal*; *SPA = Stochastic Processes and its Applications*;

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Vsevolod Malinovskii