REFLEXIVITY IN COMPETITION-ORIGINATED UNDERWRITING CYCLES

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Abstract

There seems to exist much similarity between trends in certain competition-originated insurance cycles and the behavior of stock prices. Besides an aggressive company entering the property and liability insurance market at the high point of the cycle and slashing the price to gain an advantageous market share, the competition-originated insurance cycle is led by other participants' year-to-year competition for revenue and market share framed in the concept of reflexivity. In the framework of multi-period Lundberg-type game model of insurance process, this paper addresses quantitative analysis of certain reflexive rationales of the competition-originated insurance cycles, emphasizes such leverages as expected annual earnings related to customer's migration promptness and outlines intelligent competitive strategies.

1. Introduction

The subject of the papers Malinovskii (2009) and Malinovskii (2010) is mathematical modeling of the underwriting cycles in general insurance as they play out at the global, industry wide, rather than operating level.

This paper is a direct development of Malinovskii (2010). In the latter, concerted industry response[1] to price slashing of an aggressive company is set forth as a fundamental prerequisite of the competition-originated insurance cycles model, as rivals are aggressive and defending companies with straightforward criteria of success. Consistent with economic evidence (see e.g., Feldblum (2007) and reference therein), it was rationalized by the observation that, eager to protect their individual shares, the companies in the "rest of the market" follow the aggressive

Key words and phrases. Underwriting cycles, Reflexivity, Self-reinforcing trends, Competition, Lundberg-type model, ERS analysis, Games with imperfect theory.

[1]See Assumption 3 in Section 2 of Malinovskii (2010); see also Remark 3.1 below.

company in premiums slashing. That results in a downswing market price trend. Gradually, it deteriorates the market, rendering profitable operations impossible.

Seeking more realistic scenarios and rationales than a single company influencing the whole market, as other participants care exclusively about preserving their share (i.e., stick in "carrot and stick" incentives), paramount is year-to-year underwriters' competition for revenue and market share. It is framed in the concept of reflexivity. Reflexivity, coined by Soros (1994) to oppose, supplement and criticize the equilibrium theory on financial and stock markets, is hallmarked by a double-feedback mechanism which is at play at the times of far-from-equilibrium development. It leads to a seminal insight into the competitive markets, where[2] "there is no built-in tendency toward equilibrium: to the extent that we need stability we must introduce it by deliberate policy measures".

Bearing in mind strategic planning of a company and regulation aiming stability of the insurance market as a whole, the following questions are essential. Which forces are at work generating a downswing price trend? Fraught with clustered insolvencies, at which stage does that trend deteriorate the insurance market and becomes dangerous for the market stability as a whole? Which events on the market trigger this trend, accelerate it, or slow it down and even reverse its development? Which leverages or policy measures impact these events?

Fundamental in this endeavor is similarity between, on the one hand, insurance and, on the other hand, financial and stock markets, all being markets with perfect competition. Scrutinizing the latter, Soros (1994) noted[3] that "the stock market comes as close to meeting the criteria of perfect competition as any market: a central marketplace, homogeneous products, low transaction and transportation costs, instant communications, a large enough crowd of participants to ensure that no individual can influence market prices in the ordinary course of events, and special rules for insider transactions as well as special safeguards to provide all participants with access to relevant information ... If there is any place where the theory of perfect competition ought to be "translated into practice", it is in the stock market .... Yet there is little empirical evidence of an equilibrium or even a tendency for prices to move toward an equilibrium. The concept of an equilibrium seems irrelevant at best and misleading at worst." As for the insurance market with immanent underwriting cycles, the concept of an equilibrium calls forth as strong objections as in financial or stock markets.

It is known that insurance intermediaries of all kinds face substantial competitive pressure to seek adequate coverage for their customers at the lowest price available. The totality of insureds influences in a crucial way the pricing behavior of insurers, and both parties contribute to establishment of the annual market prices paramount in this model. Besides the competitive interplay of insurers, this paper emphasizes as fundamental the pricing behavior of policyholders (see e.g., Fitzpatrick (2004)).

The rest of the paper is arranged as follows.

Section 2 sketches the global underwriting cycle's scenario. It is a translation of Soros' rudimentary model of a boom and bust sequence in stock and financial markets, into competitive insurance market. It applies causal connections of price

[2]Quotation from Chapter 3 of Soros (1994), p. 79.
[3]Quotation from Chapter 3 of Soros (1994), p. 46–47.

sensitivity of policyholders and individual premium prices, expected annual earnings and market share of the insurers. The basic causal connections are quantified further on, in Sections 4.2, 4.3, within a rigorous Lundberg-type model.

Section 3 discusses paramount concepts of the underwriting cycle's scenario. First, these are insurer's cognitive and participating functions, a basic concept of Soros' reflexivity theory. Second, it is market price which is a basic threshold value rather than just a price prevailing on the market: insureds emigrate out of the company if the individual price exceeds the market price, and immigrate into it otherwise. The market price comes from the totality of individual prices by means of a procedure which implements preferences of insureds represented by intermediaries, such as insurance agents or brokers. This section deals with a classification of insurers (trend followers, reward-seeking trend followers) and of price trends (sharp and predictable, predictable and dispersed, etc.). It is closed by remarks about a game model of insurance process.

Section 4 is devoted to quantification of casual connections on the early stage of the underwriting cycle i.e., at the stage of unrecognized trend and beginning of a self-reinforcing process. When the market price is still highly predictable, to show mercantile interests of the individual insurers, we apply the so-called Expansion-Revenue-Solvency (ERS) analysis to the Lundberg-type collective model of the annual probability mechanism of insurance. In particular, we demonstrate the reasons of individual insurers to become trend followers. The analytical study within the model introduced in Malinovskii (2010) is supplemented with numerical illustrations.

Section 5 is devoted to ERS quantification on the stage of the underwriting cycle, when only trend followers survive as active participants. Acting everyone for oneself, they may face spasmodic behavior of the annual revenue and market share which looks a worse management result. This observation suggests the rationale for spontaneous cartel-like actions that brings us back to the topics of Sections 4–6 of Malinovskii (2010), though by aggressive $\mathfrak{A}$ and defensive $\mathfrak{D}$ companies one would mean groups of solidary individual insurers.

Section 6 contains some conclusive remarks.

2. Outline of the underwriting cycle's scenario

Let us refine the rudimentary scenario of the underwriting cycle on the competitive insurance market outlined in Section 2 of Malinovskii (2010), capturing dynamics of the boom and bust sequence in stock and financial markets. The crucial features of these dynamics are: the unrecognized trend; the beginning of a self-reinforcing process; the successful test; the growing conviction, resulting in a widening divergence between reality and expectations; the flaw in perceptions; the climax; a self-reinforcing process in the opposite direction.

This global scenario will be further combined with competitive strategic models for different companies with particular goals on the market, such as competition for co-existence and of elimination of aggressive and defending companies outlined in Malinovskii (2010).

In this section our aim is not a quantitative analysis, rather a crude sketching of the interplay of the factors to be incorporated into the rigorous model. These main factors are[4]: the individual insurer's price (P), price sensitivity of policyholders (l) related to migration rate, business expansion (λ) in terms of portfolio size, revenue (e) in terms of expected annual earnings, business solvency (ψ) in terms of ruin probability. At this stage, we shall not quantify any of the variables but only indicate direction ($\uparrow$, $\downarrow$, $\updownarrow$) or order of magnitude ($>$,$<$). We use freely the notions of market price[5] $\boldsymbol{P}$ which is the price prevailing on the market, of averaged losses $\mathsf{E}Y$, of years of soft (i.e., $\boldsymbol{P} < \mathsf{E}Y$) and hard (i.e., $\mathsf{E}Y < \boldsymbol{P}$) market[6], of downswing and upswing quarters linked in a cycle, and so on.

2.1. Cycle's scenario. To start with, on the hard market ($\mathsf{E}Y < \boldsymbol{P}$)

(i) price sensitivity of policyholders and their migration promptness substantially increases ($\uparrow l$) due to various reasons (influence of exterior economic factors, or advertising campaign of an aggressive company, or else);

(ii) as migration promptness grows sufficiently large, while the market price $\boldsymbol{P}$ is still predictable, some insurers (an aggressive company, or companies in the "rest of the market") recognize that individual price P set somewhat below $\boldsymbol{P}$ ($\downarrow P$) yields mercantile advantage in terms of increasing revenue ($\uparrow e$) and portfolio size ($\uparrow \lambda$), while P somewhat above $\boldsymbol{P}$ ($\uparrow P$) works in reverse. In the form of casual connections, this observation reads as

$$\begin{aligned} &\uparrow l,\ \downarrow P \quad \longrightarrow \quad \uparrow \lambda,\ \uparrow e,\ [\uparrow \psi,\ \text{which is a latent flaw in the} \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{participants' perception}^{7}], \\ &\uparrow l,\ \uparrow P \quad \longrightarrow \quad \downarrow \lambda,\ \downarrow e,\ [\downarrow \psi]; \end{aligned} \tag{1}$$

(iii) the individual insurers in the "rest of the market" led by mercantile interests become eager to reduce individual prices somewhat below the market price $\boldsymbol{P}$. They expect the same from other rational participants. It yields a stretch where falling prices are reinforced by a positive bias;

(iv) the enhanced downward market price trend influences the prevailing bias of individual insurers in the "rest of the market". More and more of them become trend followers ($\downarrow P$). Growing change in perceptions affects the market price $\boldsymbol{P}$ which gradually decreases, becoming less predictable. As market price $\boldsymbol{P}$ decreases, more incentives are created for price-sensitive customers to seek for advantageous price. That reduces the number of price-insensitive insureds and increases customer's migration promptness ($\uparrow l$);

(v) those who are inclined to fight the trend are progressively eliminated and in the end only trend followers survive as active participants. As customer's migration promptness is high ($\uparrow l$), the majority of reward seeking[8] trend followers recognize that to stabilize and succeed in predictable growth of

[4]In parentheses are the letters which are core of the formal notation in rigorous model of Section 4.

[5]See Definition 2.1 in Malinovskii (2010).

[6]See Definition 2.2. in Malinovskii (2010).

[7]Cf. Soros (1994), p. 44–45: "There is bound to be a flaw in the participants' perception of the fundamentals. The flaw may not be apparent in the early stages but it is likely to manifest itself later on. When it does, it sets the stage for a reversal in the prevailing bias. If the change in bias reverses the underlying trend a self-reinforcing process is set in motion in the opposite direction. What the flaw is and how and when it is likely to manifest itself are the keys to understanding the boom and bust sequences."

[8]Reward seeking are those who are eager to have persistently, year-by-year, positive business results (increasing revenue and portfolio) in exchange of slashing prices.

revenue and market share ($\uparrow e$, $\uparrow \lambda$), it is indispensable to have market price $\boldsymbol{P}$ predictable in size and variance. It results in spontaneous price cartel-like actions which means solidary actions of groups of insurers with similar parameters, and pursuing similar goals. A predictable and robust downward market price trend[9] unfolds from their perceptions and speculative actions. As speculation gains in importance, other factors lose their influence. There is nothing to guide speculators but the market itself, and the market is dominated by solidary groups of individual insurers joined the cartels. Customer's migration promptness remains high ($\uparrow l$).

As the soft market ($\boldsymbol{P} < \mathsf{E}Y$) is approached or occurs,

(vi) clustered insolvencies happen on the deteriorated market, and the individual insurers in the "rest of the market" recognize that $\downarrow P \quad \longrightarrow \quad \downarrow \psi$;
(vii) losers are progressively eliminated, survivors win spoils. Aggressive survivors are appeased;
(viii) latent flaw in the participants' perceptions mentioned in item (ii) grows culminating: solvency-conscious insurers grow fastidious, media publish information about clustered insolvencies, policyholders fear losing their insurance protection in consequence of insurer's ruin. Insureds' migration rate drops down ($\downarrow l$);
(ix) as migration rate gets exceedingly low ($\downarrow l$), the individual insurers recognize the casual connection

$$\downarrow l,\ \uparrow P \quad \longrightarrow \quad \downarrow \psi,\ \uparrow e,\ [\updownarrow \lambda \text{ since migration is very low}]. \tag{2}$$

Eventually, a crossover point would have been reached, even without the intervention of the authorities, when the inflow of speculations could not keep pace with the solvency deficit, and the trend would have been reversed,

(x) the individual insurers in the "rest of the market" led by solvency interests, become eager to increase individual prices somewhat above $\boldsymbol{P}$ ($\uparrow P$) and expect the same from the other participants. It yields a stretch where rising prices are reinforced by a positive bias;
(xi) as customer's migration promptness is low ($\downarrow l$), growing change in perceptions ($\uparrow P$) affects the market prices $\boldsymbol{P}$ which gradually increase. That results in upswing market price trend which gradually improves the market, focusing profitable operations.

Looking closer at the crossover point[10], "there are, of course, mitigating circumstances. One is that market participants are likely to recognize a change in trend only gradually. The other is that the authorities are bound to be aware of the danger and do something to prevent a crash."

2.2. Immediate comments. Make a few immediate comments about this rudimentary scenario. Most of its items need more supporting arguments, up to rigorous modeling allowing quantitative analysis specific for the insurance context.

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[9] Cf. Soros (1994), p. 77: "These three tendencies are mutually self-validating. It is the growth in annual earnings and market shares that makes the market price trend so persistent. It is the persistence of the trend that makes a trend following bias so rewarding. It is the rewards reaped by speculative slashing premiums that attract the totality of participants."

[10] Quotation from Chapter 3 of Soros (1994), p. 77.

First, throughout all this scenario, the individual insurer's efforts to understand the situation and insurer's decisions based on that understanding are alloyed. Formalized as insurer's cognitive and participating functions in Section 3, it is a central point of reflexivity analysis.

Second, since we focus on insurers' competition for revenue and market share based on migration of insureds, price sensitivity of policyholders[11] is paramount.

Bearing in mind fundamental identification of the intermediary, whether denominated as "agent" or "broker", with his customer, insurance intermediaries of all kinds face substantial competitive pressure to seek adequate coverage for their customers at the lowest price available. According to Fitzpatrick (2004), the first priority of an independent insurance agency or brokerage is to obtain the "best deal" for their customer. Put simply, whatever an agent's or brokers legal relation to the underwriting carrier, they are motivated, in practical terms, by the fear that they will lose their customer to another agent or broker who can deliver the same coverage at a lower price.

Thus, the influence of brokers in the insurance marketplace almost guarantees that underwriters will err on the side of under-pricing their products, increasing the severity of pricing spikes when the real cost of the coverage is ultimately determined. Brokers also effectively play insurance carriers against one another — indeed, they may be described as the true victors in the auction market they maintain, while only the Winner's Curse[12] awaits the "successful" insuring bidder.

Going back to scenario above, introduce the following assumption.

ASSUMPTION 1 (Consumers' seek for better prices). *On the hard market, a growing tendency to reduce prices stimulates policyholders to seek for better prices.*

In other words, on the hard market (items (ii)—(v)) self-supporting, or even self-reinforcing casual connection holds true:

$$\uparrow l \quad \leftrightarrows \quad \downarrow P. \tag{3}$$

It is noteworthy that insurer's cognitive and participating functions yield the totality of individual insurer's annual prices, and the customers represented by their intermediaries of all kinds develop finally the market price value.

Third, it is the growth in mercantile advantages moving in a market price trend following fashion that makes the trend so persistent; it is the persistence of the trend that makes a trend following bias so rewarding; and it is the rewards reaped by speculation that attract increasing amounts of capital.

Starting with causal connections of item (ii), as customer's migration promptness is sufficiently large, more premiums from immigrants may compensate or exceed losses from premium cut per policy. So, seek of revenue motivates early slashing premiums which is a bonus to a greater market share. Quantitative insight into these premises will be called Expansion-Revenue-Solvency (ERS) analysis and starts in Section 4 of this paper.

[11]Cf. Daykin et al. (1996), p. 343: "one of the relevant factors is the price sensitivity of policyholders. This obviously depends on the extent to which brokers are used and can be very different for commercial policies and personal lines policies."

[12]The Winner's Curse is an economic theory hypothesizing that the winning participants in an auction will typically pay too much for the auctioned item — or, in the insurance context, charge too little to win a customer — because the nature of an auction is to favor the bidder with the most optimistic assessment of the value of the underlying asset.

As the portion of insurers acting according to pattern of items (ii)–(iii) grows to be a considerable group in the market, overall decrease of the market price becomes apparent as an outcome of the procedure yielding market price. Item (iv) refers to a reflexive relationship in which market prices are determined by two factors — underlying trend and prevailing bias — both of which are, in turn, influenced by falling market prices.

In item (v), after the downward market price trend is established, those who are inclined to fight the trend are progressively eliminated and in the end only trend followers survive as active participants. As speculation gains in importance, other factors lose their influence. There is nothing to guide speculators but the market itself, and the market is dominated by trend followers. Of further interest is Expansion-Revenue-Solvency (ERS) insight into how the market price is yielded low spread by speculative decisions of the trend followers seeking for more reward.

This scenario agrees with some universally valid observations made by Soros (1994) about mutually self-validating tendencies in a boom and bust sequence. In particular, the relative importance of speculative transactions tends to increase during the lifetime of a self-reinforcing trend. Then, the prevailing bias is a trend following one and the longer the trend persists, the stronger the bias becomes. The third is simply that once a trend is established it tends to persist and to run its full course; when the turn finally comes, it tends to set into motion a self-reinforcing process in the opposite direction. In other words, prices tend to move in large waves, with each move lasting several years.

3. Paramount concepts of the underwriting cycle's scenario

Our ultimate goal is to quantify the cycle's scenario within a suitable model. In this section we discuss insurer's cognitive and participating functions which yield market price. It is not only the insurance price rate prevailing on the market, but a threshold price value decisive for migration of insureds: if the individual price P exceeds $\boldsymbol{P}$, insureds emigrate from the company, and immigrate into it otherwise.

3.1. Insurer's cognitive and participating functions. The connection between market participants' thinking and the situation in which they participate can be broken up into two functional relationships. Soros (1994) refers to the participants' efforts to understand the situation as the cognitive or passive function, and to the impact of their thinking on the real world as the participating or active function.

It can be seen that the two functions work in opposite directions: in the cognitive function the independent variable is the situation; in the participating function it is the participants' thinking.

The insurer's efforts to understand the situation on the market largely amount to prediction of the net year market price. Define the cognitive function of i-th insurer formalizing its perceptions about k-th year market price as follows:

$$\widehat{\boldsymbol{P}}^i_k = \mathcal{F}(\text{company's observations \& understanding of situation}), \quad i = 1, \ldots, n,\ k = 1, 2, \ldots. \tag{4}$$

By "understanding of situation" one means analytics based on access to intelligence information about different factors such as the market structure (i.e., portion of aggressive, neutral and defending insurers among n companies[13] which compose the market), the financial position of competitors since that bounds their challenges, price sensitivity and migration promptness of policyholders, and so on. These factors may be observable or not, in the former case — observed either partially or completely.

As for participating function of i-th insurer formalizing its decision about k-th year individual price, defined as

$$P_k^i = \mathcal{G}(\widehat{\boldsymbol{P}}_k^i \text{ \& company's particulars}), \quad i = 1, \dots, n, \; k = 1, 2, \dots, \tag{5}$$

it depends on participants' perceptions about the future market price $\widehat{\boldsymbol{P}}_k^i$, and on the set of participants' particulars. Having developed a judgement about $\widehat{\boldsymbol{P}}_k^i$, the i-th insurer selects k-th year individual price P_k^i bearing in mind such observable company's particulars as

- portfolio size (λ) and capital (u) at the beginning of the forthcoming insurance year which largely depend on the last-year business results,
- operational time horizon (t) reflecting business intensity; it is measured in operational rather than calendar time,
- portfolio structure[14], including company's idle capacity (C), consumer's loyalty marking the least possible portfolio size (c),
- claims distribution and claims arrival process, and so on.

These factors are interconnected. For example, because regulators and rating agencies require that liability insurers maintain certain fixed premium to capital (or surplus) ratios, capital impairments caused by extraordinary losses not otherwise reserved for will leave insurers no choice but to restrict their available capacity C until new capital can be raised.

Those are the main factors of the Lundberg-type collective model of the annual probability mechanism of insurance to be specified later.

3.2. Market price and insurance rating system. The cycle's scenario of Section 2 accentuates the interplay in which both the situation and the participants' views are dependent variables so that an initial change precipitates further changes both in the situation and in the participants' views. By participants we mean, on the one hand, the insurers and, on the other hand, the insureds represented by insurance intermediaries such as insurance agents or brokers.

Actual market price $\boldsymbol{P}$ is a reference value developed for insureds and with participation of insureds, for insurers and with participation of insurers. It may

[13] Generally speaking, the total number n of companies on the market depends on the year number k, i.e., $n = n_k$. On the one hand, new companies may enter the market after the beginning of our analysis, so n may increase. On the other hand, some companies may be ruined, so n may decrease throughout insurance years. For notation simplicity, we will not endow n_k with the subscript.

[14] Idle capacity C is related to company's technical particulars and may be regulated by management, while the least possible portfolio size c requires case studies. Quote from Subramanian (1998), p. 39: "Surveys of policyholders have consistently demonstrated some reluctance to switch insurers. In a survey of 2462 policyholders by Cummins et al. (1974), 54% of respondents confessed never to have shopped around for auto insurance prices. To the question "Which is the most important factor in your decision to buy insurance?", 40% responded the company, 29% the agent, and only 27% the premium. A similar survey of 2004 Germans (see Schlesinger et al. (1993)) indicated that, despite the fact that 67% of those responding knew that considerable price differences exist between automobile insurers, only 35% chose their carrier on the basis of their favorable premium. Therefore, we will assume that, given the opportunity to switch for a reduced premium, one-third of the policyholders will do so."

happen that no one company would assign actual prices equal to $\boldsymbol{P}$. In a sense, $\boldsymbol{P}$ is a value allowing price-sensitive insureds to evaluate their attitude to migration. For individual insurers who just have developed their evaluation of the market price, the discrepancy between the actual and predicted values yields the bias of the market and influences forthcoming cognitive and participating decisions.

Here the idea of imperfect understanding is articulated. Soros (1994) claimed that[15] "my approach is to tackle the problem of imperfect understanding head on. What makes the participants' understanding imperfect is that their thinking affects the situation to which it relates ... It is obviously not the only force shaping the course of events, but it is a force which is unique to events that have thinking participants. Hence it deserves to take center stage."

Following these ideas, the totality of insurer's decisions about k-th year individual prices yields the actual market price

$$\boldsymbol{P}_k = \mathcal{P}(P_k^1, \dots, P_k^n), \quad k = 1, 2, \dots, \tag{6}$$

by an averaging, by taking a minimal value among $P_k^1, \dots, P_k^n$, or by a procedure of another sort.

Though insurance intermediaries of all kinds face substantial competitive pressure to seek adequate coverage for their customers at the lowest price available, it does not mean that the market price bounds to be the smallest price from the set $P_k^1, \dots, P_k^n$ at all stages of the cycle's scenario. This seems the more true, the closer $\max_{1\leqslant i\leqslant n} P_k^i$ lies to $\min_{1\leqslant i\leqslant n} P_k^i$.

Remark 3.1. In Malinovskii (2010), essential is Assumption 3 which claims that aggression calls forth a concerted industry response, i.e. that as an aggressive company $\mathfrak{A}$ persistently seeks a larger market share, and reduces its prices $P_k^{\mathfrak{A}}$ below the current market price $\boldsymbol{P}_k$ over a series of insurance years $k = 1, 2, \dots$, the industry matches these prices after one year. Thus, in the years of hard market one has

$$\boldsymbol{P}_1 > P_1^{\mathfrak{A}} = \boldsymbol{P}_2 > P_2^{\mathfrak{A}} = \boldsymbol{P}_3 > \cdots > \mathsf{E}Y. \tag{7}$$

These price dynamics (7) appear to be a driving force for the cyclic movement of the whole market. As to the procedure (6), it means that $P_k^{\mathfrak{A}} < \mathcal{P}(P_k^1, \dots, P_k^n)$, where $P_k^{\mathfrak{A}} \in \{P_k^1, \dots, P_k^n\}$, and that $\mathcal{P}(P_k^1, \dots, P_k^n) \neq \min_{1\leqslant i\leqslant n} P_k^i$.

Important in (6) is that insurer's cognitive and participating functions operate at the same time, and interfere with each other. The sequence of events does not lead directly from one set of facts to the next; rather, it connects facts to perceptions and perceptions to facts in a shoelace pattern.

The averaging procedure has to be supplemented by a spread-measuring procedure

$$\boldsymbol{D}_k = \mathcal{D}(P_k^1, \dots, P_k^n). \tag{8}$$

For example, one may set

$$\mathcal{D}(P_k^1, \dots, P_k^n) = \#\{i : \min_{1\leqslant j\leqslant n} P_k^j \leqslant P_k^i < \mathcal{P}(P_k^1, \dots, P_k^n)\}, \tag{9}$$

which is the number of insurers whose individual prices fall below the actual market price.

[15]Quotation from Chapter 1 of Soros (1994), p. 40.

3.3. Price trend on different stages of cycle's scenario. Within several years a trend may be predictable and composed of dispersed individual price decisions ($\boldsymbol{P}$ in (6) is known, $\boldsymbol{D}$ in (8) is large), or predictable and sharp i.e., composed of concerted individual price decisions ($\boldsymbol{P}$ is known, $\boldsymbol{D}$ is small; that approaches the situation of complete knowledge). It may be unpredictable and dispersed ($\boldsymbol{P}$ is unknown, $\boldsymbol{D}$ is large; the case of largely chaotic market), or unpredictable and sharp ($\boldsymbol{P}$ is unknown, $\boldsymbol{D}$ is small; that may occur in the "mouton de Panurge" situation, as individuals will blindly follow others regardless of the consequences).

TABLE 3.1. Trends on the market

	sharp	dispersed
predictable	$\boldsymbol{P}$ is (fiducially) known, $\boldsymbol{D}$ is small	$\boldsymbol{P}$ is (fiducially) known, $\boldsymbol{D}$ is large
unpredictable	$\boldsymbol{P}$ is unknown, $\boldsymbol{D}$ is small	$\boldsymbol{P}$ is unknown, $\boldsymbol{D}$ is large

The following observations are sensible but require further case study.

(a) Market price trend in the years of prosperous market is flat, predictable and sharp since the majority of insurers are over the long time concerned with established profitable operations.

(b) Market price trend at the early stages of a downswing phase of the underwriting cycle, as only a few trend provokers start slashing prices, is smoothly declining and remains predictable and sharp.

(c) Market price trend, as trend followers increase in number and become dominating, is declining and moves from predictable and sharp to less predictable[16] and highly dispersed.

(d) Market price trend, as dominating trend followers become reward seeking, is declining, but moves again from less predictable and highly dispersed to highly predictable and sharp, due to spontaneous price cartel-like actions.

3.4. Reward seeking trend followers. Within the cycle's scenario of Section 2, aggressive insurer $\mathfrak{A}$ may play different roles: trend provoker, trend follower, trend supporter, or trend amplifier. Neutral insurer $\mathfrak{N}$ may be trend neglector or trend follower, or, which refers more to defensive insurer $\mathfrak{D}$, trend preventer, trend contester, trend fighter, trend opposer, trend resistant.

More and more insurers become trend followers on the early stage of the cycle (item (ii) of the cycle's scenario) since the individual price P set somewhat below the market price $\boldsymbol{P}$ yields year-by-year profitable and growing business. Price P somewhat above $\boldsymbol{P}$ works in reverse and those who resist or disregard downward price trend are progressively eliminated (item (v) of the cycle's scenario).

Seeking for year-by-year profitable and growing business remains the main mercantile rationale on the phase when the market grows dominated by trend followers. However, on this phase the market price trend switches from highly predictable and sharp, as a few insurers are trend provokers or trend supporters, to predictable but dispersed, since the procedure (6) tends to be an averaging.

On this stage the business results in terms of revenue and market share may become fluctuating rather than monotone growing, even as manager's inner attitude is a genuine trend follower's one. Spasmodic annual revenue and market share

[16]That depends on professionalism of analytic departments which watch the market.

results linked with decreasing price looks psychologically indeed a worse management. Further discussion of that observation which holds responsibility on poor perception, are put in Section 5 below.

Reward seeking are those trend followers who are eager to have persistently, year-by-year, positive business results in exchange of slashing prices, even as price trend grows less predictable or highly dispersed. Quoting Soros (1994), p. 77, "there is nothing to guide speculators but the market itself", and the wish of majority of reward seeking trend followers results in spontaneous price cartel-like actions rendering self-reinforcing price trend sharp and predictable. Emphasize it that we do not mean formal agreement, rather inconscient course of events.

To bring this subsection to a close, cite a paramount observation of Soros (1994), p. 14, concerning stock and financial markets, but appropriate in the above context: "the generally accepted view is that markets are always right — that is, market prices tend to discount future developments accurately even when it is unclear what those developments are. I start with the opposite point of view. I believe that market prices are always wrong in the sense that they present a biased view of the future. But distortion works in both directions: not only do market participants operate with a bias, but their bias can also influence the course of events. This may create the impression that markets anticipate future developments accurately, but in fact it is not present expectations that correspond to future events but future events that are shaped by present expectations."

3.5. Remarks about a game model of insurance process. The multi-period game model of the insurance process is an upgrade of a control-theoretical model developed in Sections 5, 6 of Malinovskii (2010). It describes the market consisting of n interacting insurers and the totality of policyholders.

The insurance process over the market matches the diagram (cf. Eq. (1) and (2) in Malinovskii (2010))

$$\underbrace{\mathsf{w}_0 \xrightarrow{\gamma_0} \mathsf{u}_0 \xrightarrow{\mathcal{P}_1} \boldsymbol{P}_1 \boxed{\downarrow\uparrow}\, l_1 \xrightarrow{\pi_1} \mathsf{w}_1}_{\text{1-st year; initial price sensitivity } l_0} \cdots$$

$$\cdots \xrightarrow{\pi_{k-1}} \mathsf{w}_{k-1} \underbrace{\xrightarrow{\gamma_{k-1}} \mathsf{u}_{k-1} \xrightarrow{\mathcal{P}_k} \boldsymbol{P}_k \boxed{\downarrow\uparrow}\, l_k \xrightarrow{\pi_k} \mathsf{w}_k}_{k\text{-th year; initial price sensitivity } l_{k-1}} \cdots, \quad (10)$$

where $\mathsf{w}_k = (\mathsf{w}_k^{\langle 1\rangle}, \dots, \mathsf{w}_k^{\langle n\rangle}) \in \mathrm{W}$, $\mathsf{u}_k = (\mathsf{u}_k^{\langle 1\rangle}, \dots, \mathsf{u}_k^{\langle n\rangle}) \in \mathrm{U}$, $k = 0, 1, \dots,$

$$\mathrm{W} = \mathrm{W}^{\langle 1\rangle} \times \cdots \times \mathrm{W}^{\langle n\rangle} \quad \text{and} \quad \mathrm{U} = \mathrm{U}^{\langle 1\rangle} \times \cdots \times \mathrm{U}^{\langle n\rangle}$$

are the market state and the market control spaces, $\mathrm{W}^{\langle i\rangle}$ and $\mathrm{U}^{\langle i\rangle}$ are the state and the control spaces of i-th company, $i = 1, 2, \dots, n$.

Mathematically, rigorous construction is yielded by a controlled random sequence $(\mathbf{W}_k, \mathbf{U}_k)$, $k = 0, 1, 2, \dots,$ which is similar to one in Malinovskii (2010).

By n-tuples $\gamma_{k-1} = (\gamma_{k-1}^{[1]}, \dots, \gamma_{k-1}^{[n]})$ $\pi_k = (\pi_k^{[1]}, \dots, \pi_k^{[n]})$, $k = 1, 2, \dots,$ we denote k-th year's aggregate control and k-th year's probability mechanism of insurance. These components, which apply collective approach to modeling an individual insurer, were scrutinized in Sections 3–7 of Malinovskii (2010). By superscripts $\mathcal{P}_k$ over the arrows in the diagram (10) we denote k-th year's mechanism which produces market price $\boldsymbol{P}_k$ from the totality of k-th year's individual prices P_k^i,

$i = 1, 2, \dots, n$ (see Eq. (5), (6)). The symbol $\boxed{\downarrow\uparrow}$ refers to coherence of k-th year's migration promptness of policyholders and of k-th year's market price. While k-th year's control decisions are made by the totality of insurers, their derivatives, l_k and $\boldsymbol{P}_k$, the former being largely a function of the latter, remain unknown for all decision-makers at the moment of decision.

Tempting is theoretically find a best policy for the playing of the game, and know it in all details, as discussed in von Neumann and Morgenstern (1944). Difficulty lies in the fact that optimality reads differently for different stages of the game and for interacting players with different objectives, such as aggressive or defending companies, and neutral companies in the "rest of the market".

Recall that the "rest of the market" means a majority of individual companies with largely independent management, pursuing their own's business interests, but no strategic goals such as to win or defend a leading position on the market. The aggressive company refers to insurer seeking a greater market share, whose behavior may trigger the competition-originated cycle. Having large exogenous capital, aggressor aims to seize a larger market share and to win a leading position by slashing prices. Behavior of that kind is conventional for intruders entering the market at the high point of the insurance cycle. The conservative company is typically an established businesses with a substantial share of the market seeking for profitable business, but mobilizing itself for defence if the gravity of the aggressor's threat or danger of market's decline is recognized[17].

The game model with n dissimilar players, let alone policyholders, gets extremely complicated, and we are facing a game with *imperfect theory* i.e., that one where one can not theoretically find a policy which could be on some mathematical criteria described as optimal. However, it does not deny strategic planning outlined in Sections 2, 5 and 6 of Malinovskii (2010), understood as planning actions which put the player into a favorable position.

Though there is no universally valid theory for the game with imperfect theory, the following terminology applied by Wiener (1956) is useful. The *moves* of the game are aggregate annual controls u_k, $k = 1, 2, \dots$, or n-tuples in U among which *legal moves* are those which agree with the "rules of the game", inter alia approved by supervision. The price components of the moves which guarantee ε-subsistence and α-solvency in the insurance context were analyzed quantitatively in Section 4 of Malinovskii (2010).

Each year, one move must be selected by the totality of the companies on the market according to some *normative criterion*[18] of good play. A part of criteria of good play holds throughout the game, but there are other criteria that belong to a

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[17] In the very background of defending company's strategy lies an old piece of folk wisdom, the claim, namely, that solidarity within a group protects it against an outside enemy. This point of view is expressed in many traditional maxims and stories. One example occurs in Kurosawa's film *Seven Samurai*. The leader of the samurai Kambei Shimada is urging the villagers to act together to repel the coming attack of bandits. "This is a rule of war" — he says. "Collective defence protects the individual. Individual defence destroys the individual".

[18] Market-wide criterion is n-dimensional. The components corresponding to aggressive company, defending company and companies in the "rest of the market" may be strikingly different. They are devised from their strategic goals, see Section 2.2 in Malinovskii (2010).

different stages of the game[19]. Besides a *global* criterion of success[20] and *normative* criteria of good play, there must be criteria that belong to a particular stage of the game, complimentary, or special, criteria of good play[21].

For each individual player that furnishes *figure of merit*[22] which is the rationale for *selection of the next move*. The figures of merit of the moves legally possible are compared in a somewhat arbitrary manner and that move with the largest figure of merit is chosen. This selection of the next move is not necessarily, or even usually, an optimum choice, but it is a choice, and the insurance process can go on. In totality, this yields the next move of the game.

An important stage is *evaluation of the merit* of this way of playing a game. Use of rigid table of merit (if a stratagem has worked once against an opponent, it will always work) is disadvantage. More intelligent way is *learning* (a record of past games and past plays is kept and at the end of each game or each sequence of games of a determined sort, the mechanism is put to a totally different sort of use; the figure of merit is continually being re-evaluated).

Quoting Wiener (1956), "war and business are conflicts resembling games, and as such, they may be so formalized as to constitute games with definite rules." But, unlike conflicts with a few competing participants[23], business crucially involves time-varying market influenced by the totality of participants, so that (quoting Wiener (1956) further) "economic game is liable to assume the formlessness of the croquet game" in Lewis Carrol's *Alice in Wonderland*.

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[19]To be specific, the stages of the chess game are the beginning of the game (the pieces are arranged in a way that tends to make them immobile and impotent, and a development is needed that will get them out of one another's way, both for offensive and defensive purposes), the middle of the game, the end of the game (the pieces are sparse on the board, and more difficult becomes to close with the opponent for the kill). The stages in the competition-originated insurance cycle are named in Section 2.4 of Malinovskii (2010).

[20]In contrast with chess game, in insurance it is not just elimination of the contestant. Under wise regulation commonweal must come before business competition.

[21]In chess it is generally disadvantageous to lose pieces and generally advantageous to take an opponent's piece. The player who retains his mobility and right of choice, as well as the player who secures the command of a large number of squares, is usually better off than his opponent who has been careless in these respects. The normative criteria of good play in chess may be: the command criterion, the mobility criterion, the number-of-pieces criterion. Advantageous and disadvantageous actions in the competition-originated insurance cycle were discussed in Section 2.5 of Malinovskii (2010).

[22]In chess, it may be the relative importance of the command constant, the mobility constant, and the number-of-pieces constant. For single insurance companies with mercantile interest and no strategic goals such as to win or defend a leading position on the market it may be the expected annual earnings and market share.

[23]From the strategic planning premises, duel models are cautionary and educative, emphasizing advantages of a strategy, rather than a one-step series of uncoordinated actions. It is like in the famous example in Wiener (1961), p. 172, where the role of mongoose belongs to the competitor which applies an intellectual aggressive strategy. Recall this example. "The mongoose begins with a feint, which provokes the snake to strike. The mongoose dodges and makes another such feint, so that we have a rhythmical pattern of activity on the part of the two animals. However, this dance is not static but develops progressively. As it goes on, the feints of the mongoose come earlier and earlier in phase with respect to the darts of the cobra, until finally the mongoose attacks when the cobra is extended and not in a position to move rapidly. This time the mongoose's attack is not a feint but a deadly accurate bite through the cobra's brain.

In other words, the snake's pattern of action is confined to single darts, each one for itself, while the pattern of the mongoose's action involves an appreciable, if not very long, segment of the whole past of the fight. To this extent the mongoose acts like a learning machine, and the real deadliness of its attack is dependent on a much more highly organized nervous system."

The baton passed from Wiener to Soros, the latter agrees that "it is true that market participants adjust to market prices but they may be adjusting to a constantly moving target", and emphasizes the impact from expectations on the market, as the criterion of success is influenced continuously by the totality of participants. Quoting Soros (1994), "nowhere is the role of expectations more clearly visible than in financial markets. Buy and sell decisions are based on expectations about future prices, and future prices, in turn, are contingent on present buy and sell decisions. To speak of supply and demand as if they were determined by forces that are independent of the market participants' expectations is quite misleading ... The price that determines the amounts produced and consumed is not necessarily the present price. On the contrary, market participants are more likely to be guided by future prices, either as expressed in futures markets or as anticipated by themselves."

Put simply, the imperfect understanding of the participants, as they base their decisions on an inherently imperfect understanding of the situation in which they participate, is contingent on their own decisions. The participants' efforts to understand the situation, the cognitive or passive function, impacts on the real world as the participating or active function. A "shoelace" interaction of cognitive and participating functions is the core of reflexivity.

Reflexivity changes the structure of events. The participants' expectations are in a reflexive interaction with their own decisions and results in persistent self-reinforcing trends, i.e. changes in the "fundamentals", being the rule rather than the exception. What the "fundamentals" are among the yardsticks relevant to business e.g., earnings, dividends, asset value, free cash flow, depends on the participants' judgments and is therefore subject to their bias.

In the next section, bearing in mind genesis of the concerted industry response in insurance, we address the average participants' judgments about migration promptness and reveal their connection with a mercantile factor called expected annual earning. Mercantile rationale for reflexive interaction between a participant's efforts to understand the situation and his movements as a player lowering his own premiums, yields better understanding of incentives of these moves; rising migration promptness of policyholders together with slashing prices appears a leverage for the aggressive insurer to start the downswing phase of the competition-originated cycle.

4. Expansion-Revenue-Solvency (ERS) analysis

In this section, we address mercantile rationales of a trend follower in the underwriting cycle's scenario of Section 2. We start a quantitative analysis in the premises of Lundberg-type collective model of the annual probability mechanism of insurance introduced in Section 3 of Malinovskii (2010). Most results in this and in the following section hold true under quite more general technical conditions, but we bounded ourselves by the Poisson–Exponential framework of the paper Malinovskii (2010) and will address generalizations in an other paper.

Since our concern in this section is switching from prosperous market to a downswing one, as only trend provokers start slashing prices, the shape of the price trend is assumed predictable and sharp, as in items (a), (b) of Section 3.3. It means that in this section we may let for simplicity that i-th insurer ($i = 1, 2, \dots, n$)

formalizing its perceptions about k-th year market price correctly predicts the actual market price, having $\widehat{\boldsymbol{P}}^i_k = \boldsymbol{P}_k$, $k = 1, 2, \ldots$.

4.1. Lundberg-type model with varying portfolio size. Recall (see Definition 3.1 in Malinovskii (2010)) that migration rates are a parametric family of positive continuous functions of time, such that $r_{d,l}(0) = 1$ uniformly on $l \geqslant 0$, $d > 0$, and

(i) for $d > 1$, $l > 0$ the function $r_{d,l}(s)$ is monotone decreasing in $s \geqslant 0$, and $r_{d_1,l}(s) < r_{d_2,l}(s)$ for all $s > 0$, as $d_1 > d_2 > 1$,
(ii) for $0 < d < 1$, $l > 0$ the function $r_{d,l}(s)$ is monotone increasing in $s \geqslant 0$, and $r_{d_1,l}(s) > r_{d_2,l}(s)$ for all $s > 0$, as $1 > d_2 > d_1$,
(iii) for $d = 1$ or $l = 0$, the function $r_{d,l}(s)$, $s \geqslant 0$, is identically unit.

In what follows, parameter $l \geqslant 0$ is customer's migration promptness, and parameter $d > 0$ is the price-to-market ratio. Migration rate function equals to portfolio size up to the multiple $\lambda > 0$, which is the initial portfolio size,

$$\lambda_{d,l}(s) = \lambda r_{d,l}(s), \quad s \geqslant 0. \tag{11}$$

The positive function $r_{d,l} = r_{d,l}(+\infty)$, $l \geqslant 0$, $d > 0$ is called ultimate migration rate function.

To be particular, pick up the ultimate migration rate function $r_{d,l}$ from Section 3.2 of Malinovskii (2010). That is, for the price-to-market ratio $d(P) = P/\boldsymbol{P} > 0$ select arbitrarily two constants $0 \leqslant c < 1 < C$ and construct $r_{d,l}$ such that $r_{0,l} = C$, $r_{+\infty,l} = c$, as $l > 0$. To this end, set

$$r_{d,l} = (e^{-\rho d^l} + \varrho)/(e^{-\rho} + \varrho), \quad d, l \geqslant 0, \tag{12}$$

and $\varrho = c/(C - c) > 0$, $\rho = -\ln((1 - c)/(C - c)) > 0$.

Remark 4.1 (Ultimate migration rate as migration promptness grows). It is noteworthy that the functions $r_{d,l}$ in (12) considered as functions of d are monotone convergent to the step function

$$r_d = \begin{cases} C, & 0 < d < 1, \\ 1, & d = 1, \\ c, & d > 1, \end{cases}$$

as the customer's migration promptness l grows to infinity, i.e.

$$r_{d,l} \to r_d, \quad \text{as } l \to \infty.$$

The convergence to step function r_d is essential and holds true for any sensible example of ultimate migration rate function: as migration promptness grows, just a tiny excess of P over $\boldsymbol{P}$ results in emigration up to the lower limit c, while a tiny excess of $\boldsymbol{P}$ over P results in immigration up to the upper limit C. For this reason, the constants C and c set to construct $r_{d,l}$ are referred to as *company's idle capacity* and *consumer's loyalty*.

Without lack of generality[24], set

$$r_{d,l}(s) = r_{d,l} + (1 - r_{d,l})\tau(s), \quad s \geqslant 0, \tag{13}$$

[24] Of course, the analytical form of $r_{d,l}$ may differ from Eq. (12) and selected at convenience.

where $r_{d,l}$ is as in Eq. (12) and the function $\tau(s)$, $s \geqslant 0$, refers to time speed of migration. It monotone decreases to zero, as $s \to \infty$, and $\tau(0) = 1$. For example, $\tau(s) = e^{-s}$ yields *exponential* migration rate function, $\tau(s) = (1+s)^{-k}$, $k > 0$ yields *power* migration rate function.

For $k > 0$, power migration rate function (see Eq. (9) of Malinovskii (2010))

$$r_{d,l}(s) = r_{d,l} + (1 - r_{d,l})(1+s)^{-k} = 1 - (1 - r_{d,l})(1 - (1+s)^{-k}), \quad s \geqslant 0, \tag{14}$$

yields[25] ($\Lambda_{\lambda,d}(t) = \int_0^t \lambda_{d,l}(s)ds = \lambda \int_0^t r_{d,l}(s)ds$; see Definition 3.3 in Malinovskii (2010))

$$\begin{aligned}\Lambda_{\lambda,d}(t) &= \begin{cases}\lambda t r_{d,l} + \lambda(1 - r_{d,l})\big((1+t)^{1-k} - 1\big)/(1-k), & k \neq 1,\\ \lambda t r_{d,l} + \lambda(1 - r_{d,l})\ln(1+t), & k = 1\end{cases}\\ &= \begin{cases}\lambda t - \lambda(1 - r_{d,l})\big[t - \big((1+t)^{1-k} - 1\big)/(1-k)\big], & k \neq 1,\\ \lambda t - \lambda(1 - r_{d,l})(t - \ln(1+t)), & k = 1\end{cases}\end{aligned} \tag{15}$$

and

$$\mathsf{E}R_{u,\lambda,P}(t) = u + \mathsf{E}Y(g(P) - 1)\,\Lambda_{\lambda,d(P)}(t), \tag{16}$$

where $R_{u,\lambda,P}(t)$ is the risk reserve at time t (see Eq. (13), (16) in Malinovskii (2010)) and $g(P) = P/\mathsf{E}Y$ is the price-to-cost ratio.

Definition 4.1 (Expected annual insurer's earnings). By absolute and relative expected annual insurer's earnings we mean respectively

$$E_{l,t}(P) = \mathsf{E}R_{u,\lambda,P}(t) - u = (P - \mathsf{E}Y)\,\Lambda_{d(P),\lambda}(t)$$

and

$$e_{l,t}(P) = \frac{\mathsf{E}R_{u,\lambda,P}(t)}{u} = 1 + \frac{E_{l,t}(P)}{u} = 1 + \frac{P - \mathsf{E}Y}{u}\,\Lambda_{d(P),\lambda}(t).$$

Remark that, by item (iii) of the definition of ultimate migration rate function,

$$E_{l,t}(\boldsymbol{P}) = (\boldsymbol{P} - \mathsf{E}Y)\,\lambda t, \quad E_{l,t}(\mathsf{E}Y) = 0$$

and

$$e_{l,t}(\boldsymbol{P}) = 1 + (\boldsymbol{P} - \mathsf{E}Y)\lambda t/u, \quad e_{l,t}(\mathsf{E}Y) = 1,$$

which is independent on migration.

Definition 4.2. Denote by $L_{l,t}(\boldsymbol{P})$ and $R_{l,t}(\boldsymbol{P})$ the solutions of the equation[26]

$$e_{l,t}(P) = 1 + (\boldsymbol{P} - \mathsf{E}Y)\lambda t/u, \tag{17}$$

such that $L_{l,t}(\boldsymbol{P}) < \boldsymbol{P} < R_{l,t}(\boldsymbol{P})$.

Remark 4.2 (Limit of expected annual insurer's earnings). Due to Remark 4.1, expected annual insurer's earnings $E_{l,t}(P)$ and $e_{l,t}(P)$ considered as functions of P are continuous and monotone convergent, as $l \to \infty$, to the functions

$$E^\sharp_t(P) = (P - \mathsf{E}Y)\,\Lambda^\sharp_{d,\lambda}(t) \quad \text{and} \quad e^\sharp_t(P) = 1 + \frac{P - \mathsf{E}Y}{u}\,\Lambda^\sharp_{d,\lambda}(t)$$

[25] Here and in what follows we freely omit or add subscripts u, λ, l, etc. in our notation. This notational variability should not lead to misunderstanding and the dependence on omitted subscript may be easily re-constructed.

[26] Plainly, the right hand side of Eq. (17) equals $e_{l,t}(\boldsymbol{P})$.

respectively, where $\Lambda^{\sharp}_{d,\lambda}(t)$ is defined as in Eq. (15), but with $r_{d,l}$ replaced by r_d (i.e., by c, as $d > 1$, by C, as $0 < d < 1$, and by 1, as $d = 1$).

It is noteworthy that $\Lambda^{\sharp}_{d,\lambda}(t)$ is *independent on* P. Therefore, $E^{\sharp}_t(P)$ and $e^{\sharp}_t(P)$ are *linearly* growing functions for $P \in (0, \boldsymbol{P})$ and $P \in (\boldsymbol{P}, \infty)$, with a *jump* in the point $\boldsymbol{P}$.

Introduce

$$I_1(t) = \int_0^t (1 - \tau(s))\, ds, \quad I_2(t) = t - I_1(t) = \int_0^t \tau(s)\, ds$$

and rewrite absolute expected annual insurer's earnings as

$$\begin{aligned} E_{l,t}(P) &= (P - \mathsf{E}Y)\,\lambda \int_0^t r_{d(P)}(s)\, ds = (P - \mathsf{E}Y)\,\lambda \int_0^t \big(r_{d(P)} + (1 - r_{d(P)})\tau(s)\big)\, ds \\ &= (P - \mathsf{E}Y)\,\lambda \big(t + (r_{d(P)} - 1) I_1(t)\big). \end{aligned}$$

For relative expected annual insurer's earnings, one has

$$e_{l,t}(P) = 1 + \frac{(P - \mathsf{E}Y)}{u}\,\lambda \int_0^t r_{d(P)}(t) = 1 + \frac{(P - \mathsf{E}Y)}{u}\,\lambda \big(t + (r_{d(P)} - 1) I_1(t)\big).$$

In what follows we will consider relative expected annual insurer's earnings rather than absolute expected annual insurer's earnings, bearing in mind one-to-one connection $e_{l,t}(P) = 1 + E_{l,t}(P)/u$.

4.2. ERS insight into casual connections on hard market. Address quantitative insight into a circular, or spiral, casual connection (3), i.e. on the hard market ($\mathsf{E}Y < \boldsymbol{P}$) quantify the relation

$$\uparrow l \quad \leftrightarrows \quad \downarrow P. \tag{18}$$

That is a major stretch which reinforces falling prices on the hard market. It tends to be circular as the cycle unfolds; that is, variables can serve as both cause and effect in relation to other variables.

The backward implication

$$\uparrow l \leftarrow \downarrow P$$

was rationalized as Assumption 1 in Section 2.2.

The rationale of the forward implication

$$\uparrow l \rightarrow \downarrow P$$

which is paramount in items (i)–(iii) of cycle's scenario in Section 2 lies in quantification of the implications (see Eq. (1))

$$\begin{aligned} \uparrow l,\ \downarrow P \quad &\longrightarrow \quad \uparrow \lambda,\ \uparrow e,\ [\uparrow \psi], \\ \uparrow l,\ \uparrow P \quad &\longrightarrow \quad \downarrow \lambda,\ \downarrow e,\ [\downarrow \psi], \end{aligned} \tag{19}$$

which is the essence of ERS analysis.

Set $\overline{e}_{l,t} = \max_{P \in [\mathsf{E}Y, \boldsymbol{P}]} e_{l,t}(P)$, $\underline{e}_{l,t} = \min_{P \in [\boldsymbol{P}, \infty]} e_{l,t}(P)$.

Assertion 4.1 (ERS analysis on hard market). Assume that the technical assumptions of Theorem 3.3 in the paper Malinovskii (2010) hold true. On the hard market ($\mathsf{E}Y < \boldsymbol{P}$), for the insurer with parameters $C, c, \lambda, u > 0$ and portfolio consisting of insured with migration promptness $l > 0$, at the year-end time $t > 0$, the shape of continuous functions $r_{d(P)}(t)$, $e_{l,t}(P)$ and $\boldsymbol{\psi}_{u,\lambda,P}(t)$, is as follows.

(E_1) Considered as a function of P, migration rate function $r_{d(P)}(t)$ monotone decreases from $C > 1$ to $c < 1$, as P increases to infinity, and $r_{d(\boldsymbol{P})}(t) = 1$.

(E_2) Considered as a function of l, migration rate function $r_{d(P)}(t)$
  (a) monotone increases from 1 to C for $\mathsf{E}Y < P < \boldsymbol{P}$, as l increases to infinity,
  (b) monotone decreases from 1 to c for $P > \boldsymbol{P}$, as l increases to infinity.

(R_1) There exist unique prices $P^{\dagger}_{l,t}$ and $P^{\ddagger}_{l,t}$, such that $\mathsf{E}Y \leqslant P^{\dagger}_{l,t} \leqslant \boldsymbol{P} \leqslant P^{\ddagger}_{l,t}$ and $e_{l,t}(P^{\dagger}_{l,t}) = \overline{e}_{l,t}$, $e_{l,t}(P^{\ddagger}_{l,t}) = \underline{e}_{l,t}$, and relative expected annual insurer's earnings $e_{l,t}(P)$ considered as a function of P
  (a) monotone increases, as P increases, for $\mathsf{E}Y \leqslant P \leqslant P^{\dagger}_{l,t}$,
  (b) monotone decreases, as P increases, for $P^{\dagger}_{l,t} \leqslant P \leqslant P^{\ddagger}_{l,t}$,
  (c) monotone increases, as P increases, for $P > P^{\ddagger}_{l,t}$.

(R_2) $e_{l,t}(\mathsf{E}Y) = 1 < e_{l,t}(\boldsymbol{P}) = 1 + (\boldsymbol{P} - \mathsf{E}Y)\lambda t/u$.

(R_3) for l, t sufficiently large $L_{l,t}(\boldsymbol{P})$ exists and is unique; $e_{l,t}(P) \geqslant e_{l,t}(\boldsymbol{P})$ for $P \in [L_{l,t}(\boldsymbol{P}), \boldsymbol{P}]$ and

$$L_{l,t}(\boldsymbol{P}) \to \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/C, \quad \text{as } l, t \text{ increase to infinity.}$$

(R_4) for l, t sufficiently large $R_{l,t}(\boldsymbol{P})$ exists and is unique; $e_{l,t}(P) \leqslant e_{l,t}(\boldsymbol{P})$ for $P \in [\boldsymbol{P}, R_{l,t}(\boldsymbol{P})]$ and

$$R_{l,t}(\boldsymbol{P}) \to \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/c, \quad \text{as } l, t \text{ increase to infinity.}$$

(R_5) Considered as a function of l, relative expected annual insurer's earnings $e_{l,t}(P)$
  (a) monotone increases, as l increases, for $\mathsf{E}Y < P < \boldsymbol{P}$,
  (b) monotone decreases, as l increases, for $P > \boldsymbol{P}$.

(S_1) Considered as a function of P, probability of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ monotone decreases to zero, as P increases to infinity[27].

(S_2) Considered as a function of l, probability of ruin $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t)$
  (a) monotone increases, as l increases, for $\mathsf{E}Y < P < \boldsymbol{P}$,
  (b) monotone decreases, as l increases, for $P > \boldsymbol{P}$.

Proof of Assertion 4.1. Bearing in mind fundamental Remarks 4.1 and 4.2 and continuity arguments, the monotony claims of this assertion are intuitive. Straightforward is the existence and uniqueness of the points $P^{\dagger}_{l,t}$ and $P^{\ddagger}_{l,t}$, which

[27]Recall that the essence of Theorem 3.3 in Malinovskii (2010), which requires most technical assumptions, is explicit expressions for the probability of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$. In particular, for year's index $\varkappa = \boldsymbol{P}/\mathsf{E}Y > 1$ and for $\mu = 1/\mathsf{E}Y$,

$$\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = \varkappa^{-1}\exp\{-u\mu(1-\varkappa^{-1})\} - \frac{1}{\pi}\int_0^{\pi} f_{u,\lambda,\boldsymbol{P}}(x)\,dx,$$

where

$$\begin{aligned} f_{u,\lambda,\boldsymbol{P}}(x) =& \varkappa^{-1}(1+\varkappa^{-1}-2\varkappa^{-1/2}\cos x)^{-1}\exp\big\{u\mu\big(\varkappa^{-1/2}\cos x - 1\big) - \lambda t\varkappa\big(1+\varkappa^{-1}-2\varkappa^{-1/2}\cos x\big)\big\} \\ &\times \big[\cos\big(u\mu\varkappa^{-1/2}\sin x\big) - \cos\big(u\mu\varkappa^{-1/2}\sin x + 2x\big)\big]. \end{aligned}$$

It is noteworthy that $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = \boldsymbol{\psi}_{u,\lambda,P}(t)\Big|_{P=\boldsymbol{P}}$ does not depend on migration.

are local maximum and local minimum of $e_{l,t}(P)$ in intervals $(\mathsf{E}Y, \boldsymbol{P})$ and $(\boldsymbol{P}, \infty)$ respectively, and of the points $L_{l,t} < \boldsymbol{P}$ and $R_{l,t} > \boldsymbol{P}$, which are two only solutions of the Eq. (17). Formal proofs require direct calculus and are left to the reader.

Check items (R_3) and (R_4). To find roots of Eq. (17), rewrite it as

$$(P - \mathsf{E}Y)\big(1 + (r_{d(P),l} - 1)I_1(t)/t\big) = (\boldsymbol{P} - \mathsf{E}Y)\big(1 + (r_{d(\boldsymbol{P}),l} - 1)I_1(t)/t\big).$$

It is noteworthy that this equation is independent on λ and u. Since $I_1(t)/t \to 1$ (or $I_2(t)/t = (t - I_1(t))/t \to 0$), as $t \to \infty$, one has equation for the main terms,

$$(P - \mathsf{E}Y)\big(1 + (r_{d(P),l} - 1)\big) = (\boldsymbol{P} - \mathsf{E}Y)\big(1 + (r_{d(\boldsymbol{P}),l} - 1)\big).$$

Since $d(\boldsymbol{P}) = 1$ and $r_{d(\boldsymbol{P})} = 1$, with the year's index $\varkappa = \boldsymbol{P}/\mathsf{E}Y > 1$, one has

$$\big(\varkappa\, d(P) - 1\big) r_{d(P),l} = \varkappa - 1.$$

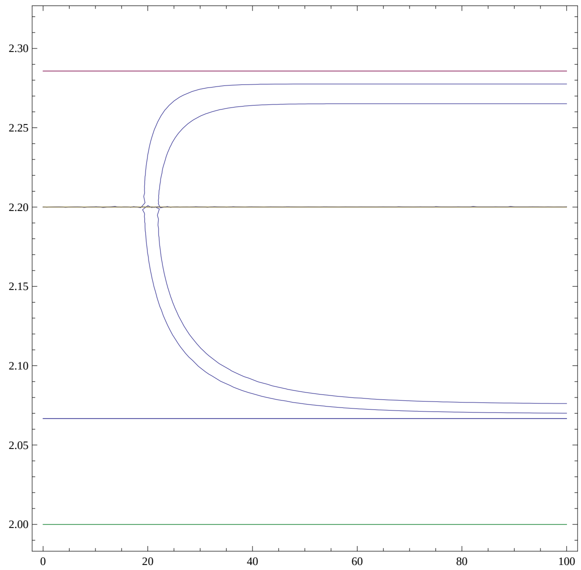

FIGURE 1. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$) with $C = 3.0$, $c = 0.7$ and $u = 35$. Lower branches: $L_{l,t}$ ($t = 100, 800$) as functions of $l \geqslant 0$. Upper branches: $R_{l,t}$ ($t = 100, 800$) as functions of $l \geqslant 0$. Horizontal lines top-down: $R(\boldsymbol{P}) = \lim_{l,t\to\infty} R_{l,t}(\boldsymbol{P}) = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/c = 2.285$, $\boldsymbol{P} = 2.2$, $L(\boldsymbol{P}) = \lim_{l,t\to\infty} L_{l,t}(\boldsymbol{P}) = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/C = 2.066$, $\mathsf{E}Y = 2.0$.

Bearing in mind Remark 4.1, one has $r_{d(P),l} \to C > 1$ for $P < \boldsymbol{P}$, and $r_{d(P),l} \to c < 1$ for $P > \boldsymbol{P}$, as $l \to \infty$. On has finally

$$L = \lim_{l,t\to\infty} L_{l,t} = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/C, \quad R = \lim_{l,t\to\infty} R_{l,t} = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/c,$$

and the proof is easily complete. It is noteworthy that L, $L_{l,t}$ $R_{l,t}$ and R are dependent on $\boldsymbol{P}$. □

Figure 1 illustrates numerically $L_{l,t}$ and $R_{l,t}$ and their limits L and R, as functions of l. It is noteworthy that the solutions $L_{l,t}$ and $R_{l,t}$ of Eq. (17) exist, as l is sufficiently large.

Remark 4.3. Formulated as a rigorous theorem within Lundberg-type collective model of the annual probability mechanism of insurance, Assertion 4.1 quantifies implications (19), paramount in the narrative scenario of Section 2. In particular, concerning the first implication (19), it yields the following.

- Assertions (E_1), (E_2) make possible to evaluate numerically the growth of portfolio ($\uparrow\lambda$), as the individual price P falls below $\boldsymbol{P}$ ($\downarrow P$), as migration promptness increases ($\uparrow l$).
- Assertions (R_1)–(R_5) make possible to evaluate numerically the growth of revenue ($\uparrow e$), as migration promptness l increases ($\uparrow l$) and the individual price P falls "somewhat below" $\boldsymbol{P}$ ($\downarrow P$), i.e., lies in the interval $[L_{l,t}(\boldsymbol{P}), \boldsymbol{P}]$.
- Assertions (S_1), (S_2) make possible to evaluate numerically the danger of a growing probability of ruin, as the individual price P falls below $\boldsymbol{P}$ and migration promptness l increases.

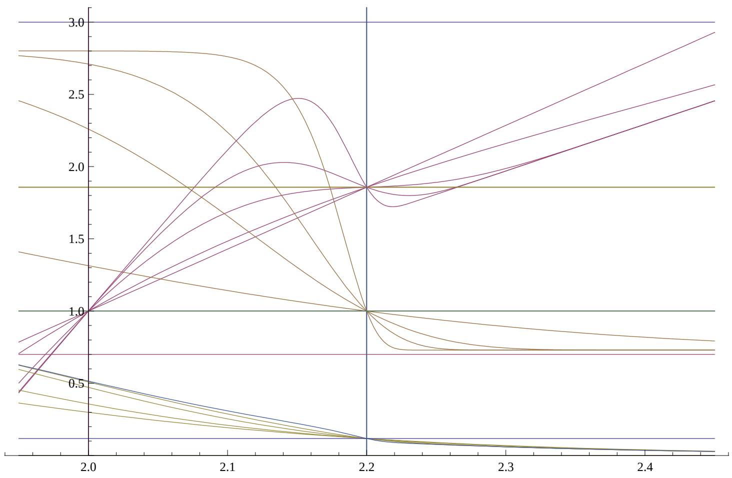

Figure 2. A company on hard market ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$), with high idle capacity $C = 3$, small capital reserve $u = 35$ and high consumer's loyalty $c = 0.7$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1-r_d)(1+t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\psi_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$, and $\lambda = 1.5$. Horizontal lines: $C = 3$, $e_{l,t}(\boldsymbol{P}) = 1.85714$, 1, $c = 0.7$, $\psi_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$.

Figures 2–9, with five former refereeing to hard market and three latter to soft market (see Section 4.3 below), deal with typical groups of insurers:

- with a high idle capacity C, high consumer's loyalty c, small initial capital reserve u and moderate initial portfolio size λ,
- with a high idle capacity C, low consumer's loyalty c, small initial capital reserve u and moderate initial portfolio size λ,
- with a high idle capacity C, low consumer's loyalty c, large initial capital reserve u and moderate initial portfolio size λ.

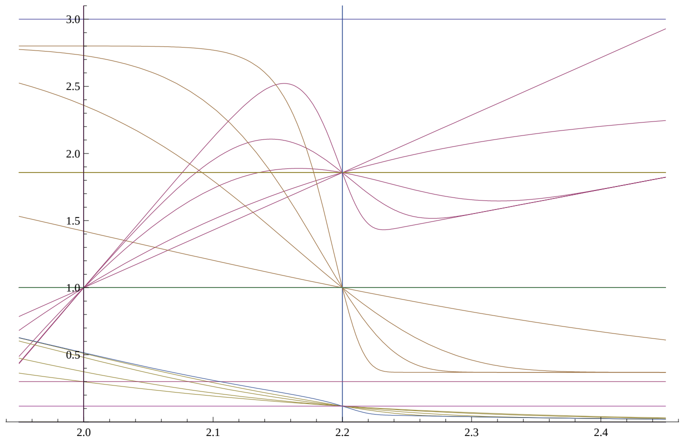

FIGURE 3. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$), with high idle capacity $C = 3$, small capital reserve $u = 35$ and low consumer's loyalty $c = 0.3$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1-r_d)(1+t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$, and $\lambda = 1.5$. Horizontal lines: $C = 3$, $e_{l,t}(\boldsymbol{P}) = 1.85714$, 1, $c = 0.3$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$.

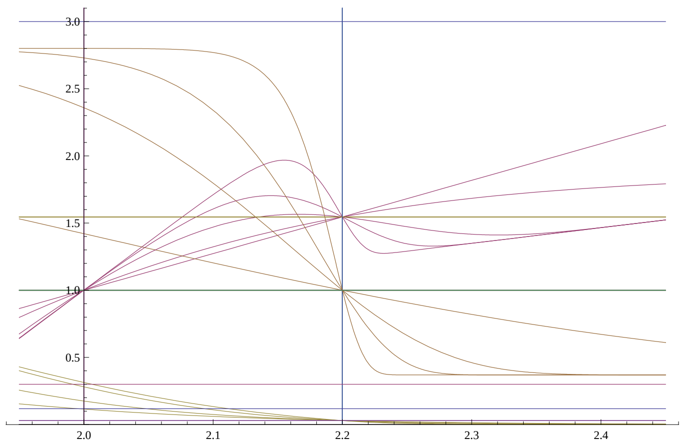

FIGURE 4. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$), with high idle capacity $C = 3$, large capital reserve $u = 55$ and low consumer's loyalty $c = 0.3$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1-r_d)(1+t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$, and $\lambda = 1.5$. Horizontal lines: $C = 3$, $e_{l,t}(\boldsymbol{P}) = 1.54545$, 1, $c = 0.3$, ruin level for smaller capital (cf. Fig. 3), $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t)\big|_{u=35} = 0.117846$, and $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t)\big|_{u=55} = 0.0298336$.

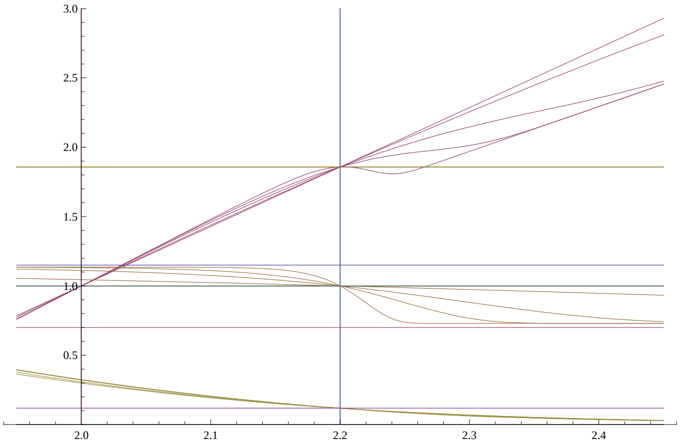

FIGURE 5. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$), with low idle capacity $C = 1.15$, small capital reserve $u = 35$ and high consumer's loyalty $c = 0.7$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1-r_d)(1+t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$, and $\lambda = 1.5$. Horizontal lines: $e_{l,t}(\boldsymbol{P}) = 1.85714$, $C = 1.15$, 1, $c = 0.7$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$.

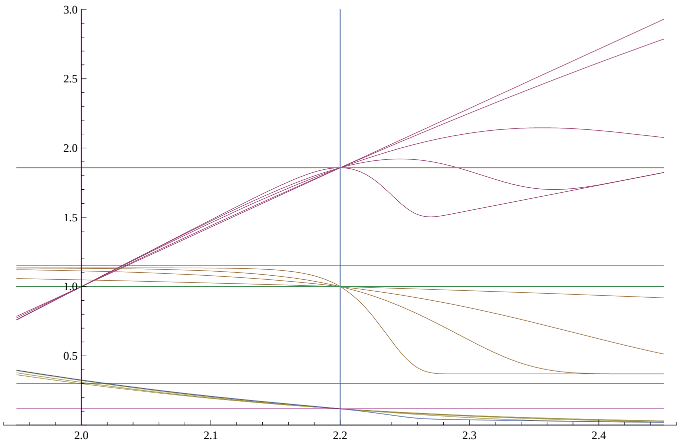

FIGURE 6. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2$), with low idle capacity $C = 1.15$, small capital reserve $u = 35$ and low consumer's loyalty $c = 0.3$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1-r_d)(1+t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$, and $\lambda = 1.5$. Horizontal lines: $e_{l,t}(\boldsymbol{P}) = 1.85714$, $C = 1.15$, 1, $c = 0.3$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$.

ERS analysis for insurers with other sets of parameters is quite analogous.

Figures 2–9 illustrate Assertion 4.1 graphically in the case of power migration rate function (14) with $k = 1/2$, as Poisson–Exponential assumptions of Theorem 3.3 in Malinovskii (2010) hold true; the latter largely required to use the explicit expression for the probabilities of ruin which particular case is footnote 27.

4.3. ERS insight into casual connections on soft market. ERS insight into casual connections on hard market ($\mathsf{E}Y < \boldsymbol{P}$) focuses on large migration promptness ($\uparrow l$). On the contrary, on soft market i.e., as $\boldsymbol{P} < \mathsf{E}Y$, focus is on low, approaching zero, migration promptness ($\downarrow l$). Paramount observation is that in this case $e_{l,t}(P)$ growth is nearly *linear*.

Items (x) and (xi) in scenario of Section 2, as insureds' migration rate drops down nearly to zero and individual price P rises somewhat above $\boldsymbol{P}$ ($\uparrow P$), deal with the casual connection (2), i.e. on the soft market ($\mathsf{E}Y < \boldsymbol{P}$)

$$\downarrow l, \uparrow P \quad \longrightarrow \quad \downarrow \psi, \uparrow e, [\updownarrow \lambda \text{ since migration is very low}]. \tag{20}$$

Quantification of (20) in the framework of Lundberg-type collective model of the annual probability mechanism of insurance is quite analogous to Assertion 4.1 and is illustrated in Fig. 7–9. By lack of space, we left details to the reader.

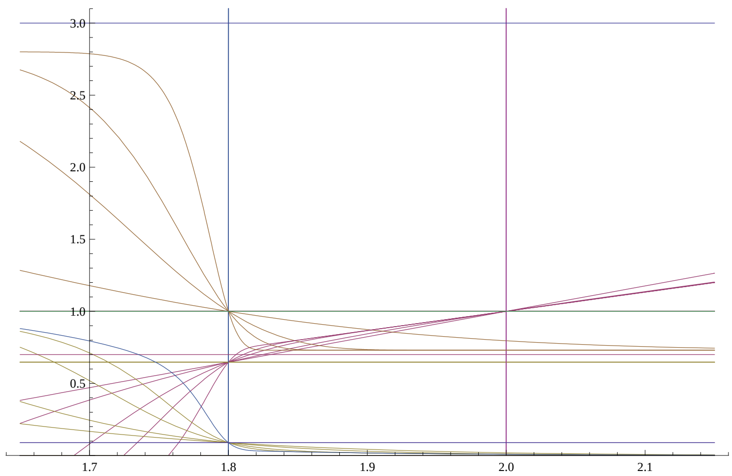

FIGURE 7. A COMPANY ON SOFT MARKET ($\boldsymbol{P} = 1.8$, $\mathsf{E}Y = 2$), with high idle capacity $C = 3$, small portfolio $\lambda = 1.5$, large capital reserve $u = 85$ and high consumer's loyalty $c = 0.7$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1 - r_d)(1 + t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$. Horizontal lines: $C = 3$, 1, $c = 0.7$, $e_{l,t}(\boldsymbol{P}) = 0.647059$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.0885231$.

5. ERS analysis for reward seeking trend followers

Consider hard market dominated by reward seeking trend followers, as customer's migration promptness is high (see items (iv), (v) of the cycle's scenario).

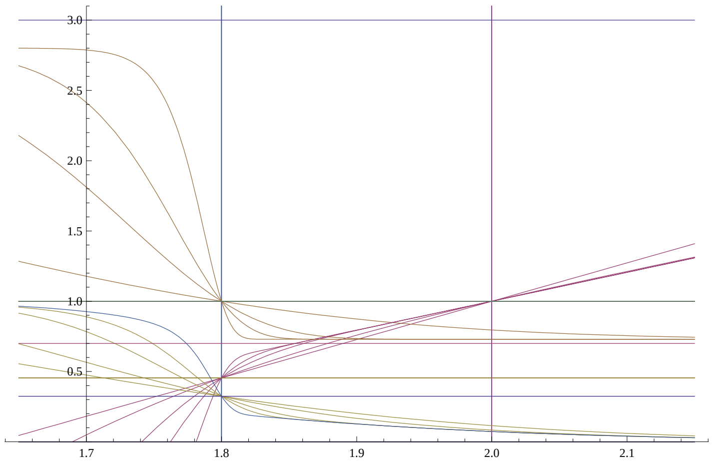

FIGURE 8. A COMPANY ON SOFT MARKET ($\boldsymbol{P} = 1.8$, $\mathsf{E}Y = 2$), with high idle capacity $C = 3$, small portfolio $\lambda = 1.5$, moderate capital reserve $u = 55$ and high consumer's loyalty $c = 0.7$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1 - r_d)(1 + t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$. Horizontal lines: $C = 3$, 1, $c = 0.7$, $e_{l,t}(\boldsymbol{P}) = 0.454545$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.324247$.

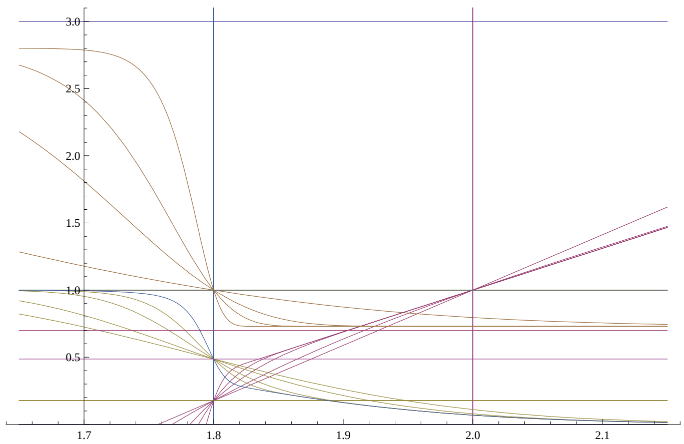

FIGURE 9. A COMPANY ON SOFT MARKET ($\boldsymbol{P} = 1.8$, $\mathsf{E}Y = 2$), with high idle capacity $C = 3$, large portfolio $\lambda = 3.5$, large capital reserve $u = 85$ and high consumer's loyalty $c = 0.7$. Shown are relative expected year-end earnings $e_{l,t}(P)$ (Y-axis; upper bundle), year-end migration rates $r_d(t) = r_d + (1 - r_d)(1 + t)^{-1/2}$ (Y-axis; middle bundle) and probabilities of ruin $\boldsymbol{\psi}_{u,\lambda,P}(t)$ (Y-axis; lower bundle) as functions of premium P (X-axis), with customer's migration promptness factor $l = 0$ (straight line), $l = 5$, $l = 20$, $l = 40$, $l = 100$ (largest oscillation), as $t = 100$. Horizontal lines: $C = 3$, 1, $c = 0.7$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.486424$, $e_{l,t}(\boldsymbol{P}) = 0.176471$.

First, show that selection of k-th year individual price P_k^i smaller than k-th year market price perceptions $\widehat{\boldsymbol{P}}_k^i$, assumed as a main individual competition leverage by all or most of (among $i = 1, 2, \ldots, n$) reward seeking insurers , will overbalance the market price trend making it more unpredictable and dispersed rather than satisfy mercantile interests of the individual trend followers. They will face spasmodic behavior of annual revenue and market share which looks a worse management result.

Second, we touch upon cartel-like actions which means solidary actions of groups of insurers with similar parameters, and pursuing similar goals. That brings us back to the topics of Sections 4–6 of Malinovskii (2010), though by e.g., aggressive $\mathfrak{A}$ and defensive $\mathfrak{D}$ companies one would rather mean solidary groups of individual insurers of these types.

5.1. Rewarding and punishing prices. For i-th insurer ($i = 1, 2, \ldots, n$), consider k-th year market price less than insurer's perceptions about k-th year market price

$$\boldsymbol{P}_k < \widehat{\boldsymbol{P}}_k^i. \tag{21}$$

Rationale for (21) is a presumption that, as all insurers evaluate approximately the same $\widehat{\boldsymbol{P}}_k^i$ and apply individual prices $P_k^i = \widehat{\boldsymbol{P}}_k^i - \Delta_k^i$ with approximately the same deductible $\Delta_k^i > 0$, the market price $\boldsymbol{P}_k = \mathcal{P}(P_k^1, \ldots, P_k^n)$ yielded by the totality of individual prices will satisfy (21), whichever a sensible procedure $\mathcal{P}$ (averaging, taking a value close to minimal, etc.) may be.

Assertion 5.1. On the hard market ($\mathsf{E}Y < \boldsymbol{P}$), for l, t sufficiently large $L_{l,t}(\boldsymbol{P})$ considered as a function of $\boldsymbol{P}$ grows, as $\boldsymbol{P}$ grows.

Proof of Assertion 5.1. It follows from item (R_3) of Assertion 4.1 which yields approximation of $L_{l,t}(\boldsymbol{P})$ by linear function $\mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/C$, as l, t increase to infinity. □

Besides the inequality $\boldsymbol{P} < \widehat{\boldsymbol{P}}$ for true *a posteriori* market price and *a priori* perseptions, and the inequality $L_{l,t}(\boldsymbol{P}) < L_{l,t}(\widehat{\boldsymbol{P}})$ of Assertion 5.1, assume without loss of generality that $L_{l,t}(\widehat{\boldsymbol{P}}) < \boldsymbol{P}$, so that

$$L_{l,t}(\boldsymbol{P}) < L_{l,t}(\widehat{\boldsymbol{P}}) < \boldsymbol{P} < \widehat{\boldsymbol{P}}.$$

It is easily seen from item (R_3) of Assertion 4.1 (see illustration in Fig. 11) that the insurer who will set the individual price P less than $\widehat{\boldsymbol{P}}$ expects to have a surplus over the level $e_{l,t}(\widehat{\boldsymbol{P}}) = 1 + (\widehat{\boldsymbol{P}} - \mathsf{E}Y)\lambda t/u$. Indeed, as $\widehat{\boldsymbol{P}}$ is the market price, one will enjoy $e_{l,t}(P) \geqslant e_{l,t}(\widehat{\boldsymbol{P}})$ for $P \in [L_{l,t}(\widehat{\boldsymbol{P}}), \widehat{\boldsymbol{P}}]$.

However, as the true market price $\boldsymbol{P}$ is smaller than $\widehat{\boldsymbol{P}}$, the reference level is $e_{l,t}(\boldsymbol{P}) = 1 + (\boldsymbol{P} - \mathsf{E}Y)\lambda t/u < e_{l,t}(\widehat{\boldsymbol{P}}) = 1 + (\widehat{\boldsymbol{P}} - \mathsf{E}Y)\lambda t/u$ and a surplus over this (smaller than expected) level will be only for $P \in [L_{l,t}(\boldsymbol{P}), \boldsymbol{P}]$.

Among the prices $P \in [L_{l,t}(\widehat{\boldsymbol{P}}), \widehat{\boldsymbol{P}}]$, call rewarding the prices

$$P \in [L_{l,t}(\widehat{\boldsymbol{P}}), \boldsymbol{P}] = [L_{l,t}(\boldsymbol{P}), \boldsymbol{P}] \cap [L_{l,t}(\widehat{\boldsymbol{P}}), \widehat{\boldsymbol{P}}]. \tag{22}$$

and punishing the prices

$$P \in [\boldsymbol{P}, \widehat{\boldsymbol{P}}]. \tag{23}$$

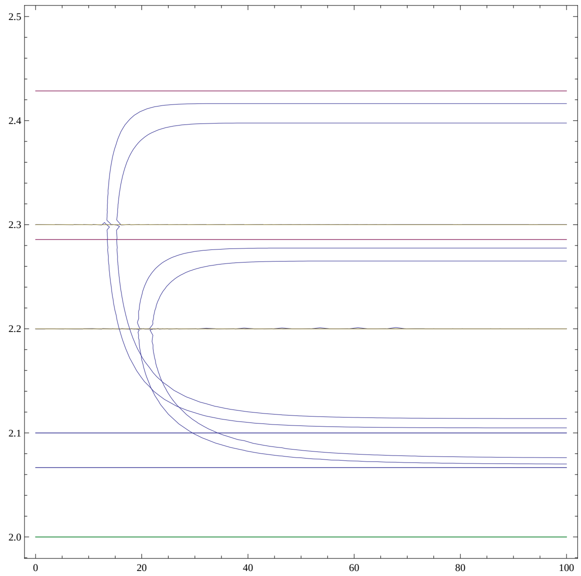

Figure 10. A company on hard market ($\mathsf{E}Y = 2$, $2.2 = \boldsymbol{P} < \widehat{\boldsymbol{P}} = 2.3$) with $C = 3.0$, $c = 0.7$, $\lambda = 1.5$ and $u = 35$. Lower branches: $L_{l,t}$ $(t = 100, 800)$ as functions of $l \geqslant 0$. Upper branches: $R_{l,t}$ $(t = 100, 800)$ as functions of $l \geqslant 0$. Horizontal lines top-down: $R(\widehat{\boldsymbol{P}}) = \lim_{l,t\to\infty} R_{l,t}(\widehat{\boldsymbol{P}}) = \mathsf{E}Y + (\widehat{\boldsymbol{P}} - \mathsf{E}Y)/c = 2.429$, $\widehat{\boldsymbol{P}} = 2.3$, $R(\boldsymbol{P}) = \lim_{l,t\to\infty} R_{l,t}(\boldsymbol{P}) = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/c = 2.285$, $\boldsymbol{P} = 2.2$, $L(\widehat{\boldsymbol{P}}) = \lim_{l,t\to\infty} L_{l,t}(\widehat{\boldsymbol{P}}) = \mathsf{E}Y + (\widehat{\boldsymbol{P}} - \mathsf{E}Y)/C = 2.1$, $L(\boldsymbol{P}) = \lim_{l,t\to\infty} L_{l,t}(\boldsymbol{P}) = \mathsf{E}Y + (\boldsymbol{P} - \mathsf{E}Y)/C = 2.066$, $\mathsf{E}Y = 2.0$.

It is noteworthy that by the very nature of the procedure $\mathcal{P}$ (averaging, taking a value close to minimal, etc.), at least half of the insurers' prices P, and typically even more, will lie above $\boldsymbol{P}$, being punishing. All these trend followers will not be rewarded by increasing revenue at the end of insurance year.

That seems to agree with the observation by Soros (1994) that the participants' bias finds expression both in the divergence between outcome and expectations and in the actual course of events.

Remark 5.1. The opposite case of $\widehat{\boldsymbol{P}} < \boldsymbol{P}$ is not typical for downswing hard market dominated by trend followers. However, we illustrate it in Fig. 12.

5.2. Cartel-like actions. An old piece of folk wisdom is: "collective defence protects the individual; individual defence destroys the individual". When all insurers became reward seeking trend followers and each practices the "individual defence", bad is the best.

Contrariwise, an *unus pro omnibus, omnes pro uno* attitude in certain groups of insurers may refurbish the market price trend to become predictable and sharp, as in Section 4. That fixes the matter for most reward seeking trend followers.

It is noteworthy however that impossible is to satisfy every insurer on the market. To think of oneself as a reward seeking trend follower does not mean yet

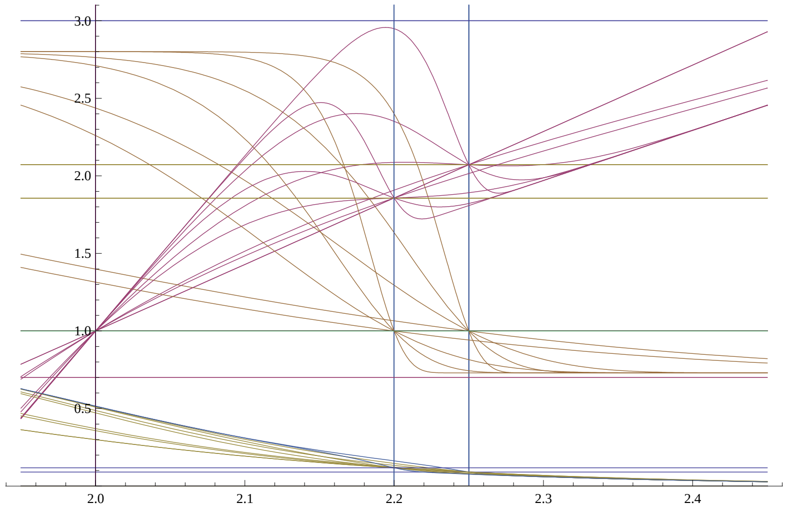

FIGURE 11. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\boldsymbol{P} = 2.2 < \widehat{\boldsymbol{P}} = 2.25$), with high idle capacity $C = 3$, small capital reserve $u = 35$ and low consumer's loyalty $c = 0.3$, $t = 100$, $\lambda = 1.5$. Horizontal lines: $C = 3$, $e_{l,t}(\widehat{\boldsymbol{P}}) = 2.07143$, $e_{l,t}(\boldsymbol{P}) = 1.85714$, 1, $c = 0.3$, $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$, and $\boldsymbol{\psi}_{u,\lambda,\widehat{\boldsymbol{P}}}(t) = 0.090647$.

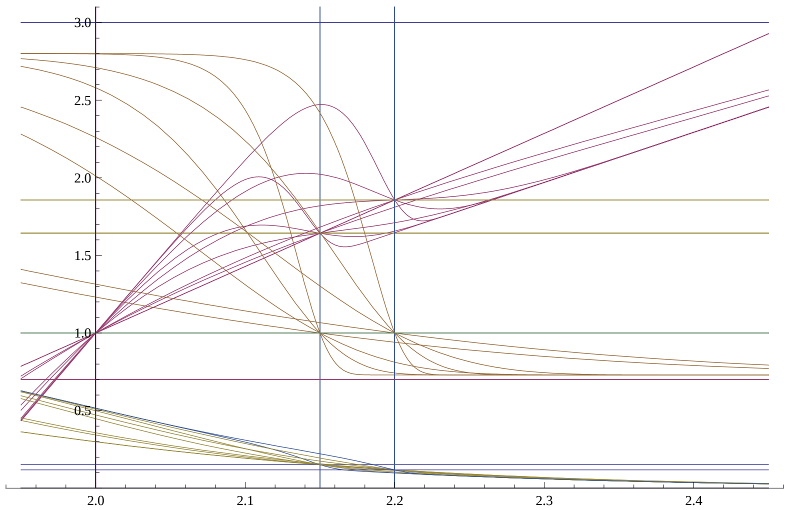

FIGURE 12. A COMPANY ON HARD MARKET ($\mathsf{E}Y = 2$, $\widehat{\boldsymbol{P}} = 2.15 < \boldsymbol{P} = 2.2$), with high idle capacity $C = 3$, small capital reserve $u = 35$ and low consumer's loyalty $c = 0.3$, $t = 100$, $\lambda = 1.5$. Horizontal lines: $C = 3$, $e_{l,t}(\boldsymbol{P}) = 1.85714$, $e_{l,t}(\widehat{\boldsymbol{P}}) = 1.64286$, 1, $c = 0.3$, $\boldsymbol{\psi}_{u,\lambda,\widehat{\boldsymbol{P}}}(t) = 0.151711$ and $\boldsymbol{\psi}_{u,\lambda,\boldsymbol{P}}(t) = 0.117846$.

to be a rewarded trend follower[28], the more so as some insurers are out of the race and forced to stop slashing prices because of growing insolvency.

[28] In spite of the French proverb *vouloir c'est pouvoir*.

6. Some conclusive remarks

6.1. Is the market always right? Most observations by Soros (1994) concerning markets with perfect competition seem to be applicable to insurance market. In particular, the point of view that "the market is always right" should be replaced by a totally opposite point of view. As Soros for stock and financial markets[29], we do not accept the proposition that market prices are a passive reflection of underlying values, nor do we accept the proposition that the reflection tends to correspond to the underlying value.

It seems that market valuations are always distorted; moreover — and this is the crucial departure from equilibrium theory — the distortions can affect the underlying values. Market prices are not merely passive reflections; they are active ingredients in a process in which both market prices and the fortunes of the insurance companies are determined. In other words, we regard changes in market prices as part of a historical process and focus on the discrepancy between the participants' expectations and the actual course of events as a causal factor in that process.

Thus we replace the assertion that markets are always right with two others:

- Markets are always biased in one direction or another.
- Markets can influence the events that they anticipate.

The combination of these two assertions explains why markets may so often appear to anticipate events correctly.

6.2. How one becomes a trend follower: carrot and stick. Quantitative analysis substantiates the observation that one becomes a trend follower due to a "carrot and stick" situation on the market. It offers a combination of rewards and punishment to induce a common insurer's behavior: besides rewards of a successful trend follower, punishment of that who is inclined to fight the trend progressively eliminates the latter, and in the end of this phase only trend followers survive as active participants. Aggressive and conservative companies may have other incentives, but the "carrot and stick" for "the rest of the market" is paramount.

One may say, bearing in mind gradual deterioration of the insurance market, that this usage of idiom "carrot and stick" is erroneous, and that in fact a "carrot on a stick" is more appropriate: the harmony of everlasting profit, expansion and solvency is eventually as out of particular insurer's reach as the carrot would always remain out of reach of the donkey moved forward to get it.

6.3. Some behavioral aspects behind the cycles. It seems that behavioral explanation of the insurance cycles yielded by Fitzpatrick (2004) agrees well with the ERS analysis of this paper.

According to Fitzpatrick (2004), "a disconnect between the incentives provided to underwriters and the long-term interest of the insurer (and its capital providers) in generating *profitable* premium growth is a key element in creating market cycles. Many companies seek to mitigate this tension by designing long-term incentive compensation plans for underwriters that are tied to profitability, but such speculative potential compensation does little to motivate the vast majority of underwriters.

[29] Below we nearly quote from Soros (1994), p. 49.

First, underwriters — like most people — are more sensitive to short-term incentives (Will I get a year-end bonus? Will a poor annual review cost me that promotion?) than they are to more speculative, deferred benefits. Moreover, the structure of the employment market in property-casualty insurance provides regular opportunities for "good producers" to move from company to company in search of greener financial pastures. In fact, the absence of significant barriers to entry in the insurance market makes for a robust employment environment and all but guarantees that an underwriter can parlay a talent for short-term premium production into a series of ever higher paying jobs at different companies.

Thus, short-term incentives to produce top-line growth and a "sellers'" job market combine to ensure that few underwriters in long-tail lines stay in one job long enough to suffer for, or even learn from, their past mistakes.

Cynics in the insurance industry call this the "write and run" phenomenon. More serious, however, is the recognition that George Santayana's observation[30] that "those who cannot remember the past are condemned to repeat it" might well have been coined to describe the insurance market."

Acknowledgments. The author is grateful to Jim Gordon for substantial improvement of the text of this paper and for his valuable comments.

References

Cummins, D., McGill, D., Winklevoss, H., Zelten, H. (1974) *Consumer Attitudes Toward Auto and Homeowners Insurance.* Philadelphia, Pa.: Department of Insurance, Wharton School.

Daykin, C.D., Pentikäinen, T., Pesonen, M. (1996) *Practical Risk Theory for Actuaries.* Chapman and Hall, London, etc.

Feldblum, S. (2007) Underwriting cycles and insurance solvency. Casualty Actuarial Society Discussion Paper Program Casualty Actuarial Society–Arlington, Virginia 1992: May, Vol. 1, 383–438, http://www.casact.org/pubs/dpp/dpp92/92dpp383.pdf

Fitzpatrick, S.M. (2004) Fear is the key: a behavioral guide to underwriting cycles. *Connecticut Insurance Law Journal*, 10, No. 2, pp. 255-275.

Malinovskii, V.K. (2009) Scenario analysis for a multi-period diffusion model of risk, *ASTIN Bulletin*, 39, 649–676.

Malinovskii, V.K. (2010) Competition-originated cycles and insurance strategies, *ASTIN Bulletin*, 40, 797–843.

von Neumann, J., Morgenstern, O. (1944) *Theory of games and economic behavior.* Princeton University Press.

Schlezinger, H., von der Schulenburg, J.M. (1993) Consumer information and decisions to switch insurers, *Journal of Risk and Insurance*, vol. LV, no. 4, p. 591–615.

Subramanian, K. (1998) Bonus-Malus systems in a competitive environment, *North American Actuarial Journal*, vol. 2, 38–44.

Soros, G. (1994) *The Alchemy of Finance: Reading the Mind of the Market.* John Wiley & Sons, Inc. New York, etc.

Wiener, N. (1961) *Cybernetics, or Control and Communication in the Animal and the Machine.* Second edition, The MIT Press and Wiley, N.Y. 1961.

[30] George Santayana, The Life of Reason: Introduction and Reason in Common Sense (Charles Scribners Sons 2d ed. 1905).

Wiener, N. (1956) *God and Golem, Inc.: A Comment on Certain Points where Cybernetics Impinges on Religion*, MIT Press, 1956, 99 p.

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