

Corrected Normal Approximation for the Probability of Ruin within Finite Time

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A new second-order approximation for the probability of ruin before time t in the framework of Andersen's risk model is suggested. This approximation is proved to be a refinement of the classical normal-type approximation and is deduced from von Bahr's representation of ruin probability in terms of ladder height distributions. The proof is based on the use of a technique developed in Malinovskii (1993) and designed for the analysis of stopped random sequences which allow the embedding of a blockwise structure. *Key words:* Andersen model, finite time ruin probability, asymptotic expansions, ladder height distributions.

1. INTRODUCTION

Much attention in collective risk theory has been given to the study of the ruin probability within finite or infinite time in the model, when the epochs of claims form a Poisson process, and in the more general Andersen's model, when such epochs form a renewal process (see e.g., Thorin (1982)). We shall consider here this more general model of risk business and derive the refinement of the classical approximation for such probabilities in the ruin problem as initial risk reserve u increases.

Even in the Poisson case the explicit expressions for such probabilities can usually only be found in terms of double Laplace transforms and their numerical evaluation requires much efforts (see e.g., Seal (1974)).

Due to this fact, there has been considerable interest in the literature in asymptotic expressions for the ruin probabilities. There exist now many different approximations, and even many different approaches to the derivation of such approximations. One is based on the fact remarked first in the Poisson case by Segerdahl (1955): the time to ruin, given finite, is asymptotically normal as $u \rightarrow \infty$. In conjunction with Cramér-Lundberg approximation for the probability of ultimate ruin this allows one to obtain a normal-type approximation for the probability of ruin within finite time (see e.g., (4.2) in Asmussen (1984)). This result was generalized to renewal arrivals of claims by von Bahr (1974). Second, diffusion approximation of the risk reserve process was used to construct another, diffusion-type approximation for the probability of ruin (see Grandell (1991) and references there).

The approximation for the ruin probability within time t as $t \rightarrow \infty$ was obtained first by Cramér (1955). A number of approximations for the probability of ruin