Scand. Actuarial J. 1996; 2: 124–147 ORIGINAL ARTICLE

Approximations and Upper Bounds on Probabilities of Large Deviations in the Problem of Ruin Within Finite Time

VSEVOLOD K. MALINOVSKII[1]

Malinovskii V. K. Approximations and upper bounds on probabilities of large deviations in the problem of ruin within finite time. Scand. Actuarial J. 1996; 2: 124–147.

In the framework of Andersen's risk model, a new asymptotic expression and upper bounds on probabilities of ruin after time $t(u) \gg \mu_T \mu_X^{-1} u$ and before time $0 < t(u) \ll \mu_T \mu_X^{-1} u$, as the initial risk reserve u increases to infinity, are suggested. This result complements the classical normal-type approximation for the probability of ruin within finite time and is designed as its large deviations counterpart. The main technical device of the paper (see Section 3), which is of independent interest, are the upper bounds and the asymptotic expressions for the probabilities of large deviations of the stopped random walks, developed under low moment conditions. *Key words: Anderson model, finite time ruin probability, large deviations, low moment conditions.*

1. INTRODUCTION

Much attention in collective risk theory has been given to the asymptotic analysis of the ruin probability $\psi(t, u)$ within finite time t, as the initial risk reserve u tends to infinity. Most interesting here is Andersen's risk model, where the epochs of claims form a general renewal process. It is the generalization of the "standard" Poisson model, where the interclaim times distributions are exponential.

There exist several types of approximations for the probabilities of ruin in the framework of Andersen's risk model. Using here and in what follows in this Section the notation introduced in Section 2 below, recall the normal-type approximation

$$\psi(t, u) \cong C\, e^{-\varkappa u} \Phi_{(m_1 u,\, D_1^2 u)}(t), \tag{1.1}$$

uniform in $t \geqslant 0$, obtained by von Bahr (1974). Malinovskii (1994) obtained the Edgeworth-type refinement of this approximation,

$$\psi(t, u) \cong C\, e^{-\varkappa u} [\Phi_{(m_1 u,\, D_1^2 u)}(t) - Q_1((t - m_1 u)/(D_1 u^{1/2}))\varphi_{(m_1 u,\, D_1^2 u)}(t)]. \tag{1.2}$$

In the Poisson/Exponential case, the approximation (1.2) was shown (see Section 3 in Malinovskii (1994)) to coincide with one proposed especially for this particular case by Asmussen (1984).

[1] The research described in this publication was supported in part by Grant No. JB7100 from the International Science Foundation and Russian Government.