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Adaptive control strategies and dependence of finite time ruin on the premium loading

Vsevolod K. Malinovskii*

Finance Academy, 125468, Leningradsky Prosp., 49, Moscow, Russia
Steklov Mathematical Institute, 119991, Gubkina Street, 8, Moscow, Russia

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Abstract

The paper is devoted to risk theory insight into the problem of asset–liability and solvency adaptive management. Two adaptive control strategies in the multiperiodic insurance risk model composed of chained classical risk models are introduced and their performance in terms of probability of ruin is examined. The analysis is based on an explicit expression of the probability of ruin within finite time in terms of Bessel functions. The dependence of that probability on the premium loading, either positive or negative, is a basic technical result of independent interest.

Keywords: Multiperiodic insurance risk model; Equity–solvency balance; Adaptive control; Classical risk model; Finite time ruin probabilities; Bessel functions; Premium loading

1. Introduction

The insurance industry is subject to intensive regulation. Supervision authorities watch compliance with the regulatory principles designed to balance the solvency and equity requirements. According to the regulatory principles, each insurer must report his financial position yearly or even more often, if required.

The insurance process is viewed therefore as a series of successive insurance years. Each year starts with a manager's control intervention which fine-tunes tariffs, reserves and other operational characteristics of the probability mechanism of insurance. Its influence remains in force throughout the whole insurance year, i.e., until the next report and subsequent control intervention.

The insurance regulation and supervision would be blind without a comprehensive model, or a set of models, describing the probability mechanism of insurance within an operating period. Suitable is the Lundberg's collective risk model which considers the net result of the risk business of an insurer from the position of a "remote" observer, and is often named the main achievement of the 20th century risk theory.

The background of the present paper is a general multiperiodic controlled risk model introduced in Malinovskii (2003) and composed of chained single-periodic risk models. The trajectory of a general multiperiodic insurance process with annual accounting and subsequent annual control may be diagrammed as

$$\underbrace{w_0 \xrightarrow{\gamma_0} u_0 \xrightarrow{\pi_1} w_1}_{\text{1st year}} \cdots \xrightarrow{\pi_{k-1}} \underbrace{w_{k-1} \xrightarrow{\gamma_{k-1}} u_{k-1} \xrightarrow{\pi_k} w_k}_{k\text{th year}} \cdots .$$

According to this diagram (for $k = 1, 2, \ldots$), at the end of the $(k-1)$th year the state variable w_{k-1} is observed; it describes the insurer's position at that time. Then, at the beginning of the kth year the control rule γ_{k-1} is applied to choose the control variable u_{k-1}. Thereupon the kth year probability mechanism of insurance unfolds; the transition function of this mechanism is denoted by π_k. It defines the insurer's position at the end of the kth year.

In that framework, consider the adaptive control strategy introduced in Malinovskii (2006a). It is defined as follows. At the time of selection of the incoming year control, let t be

* Corresponding address: Finance Academy, 125468, Leningradsky Prosp., 49, Moscow, Russia.
E-mail addresses: malinov@orc.ru, malinov@mi.ras.ru, malinov@fa.ru.
URLs: http://www.mi.ras.ru/~malinov, http://www.actuaries.fa.ru.

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